School - Level

Trigonometry

Topic 1: Trigonometric (Circular Functions)

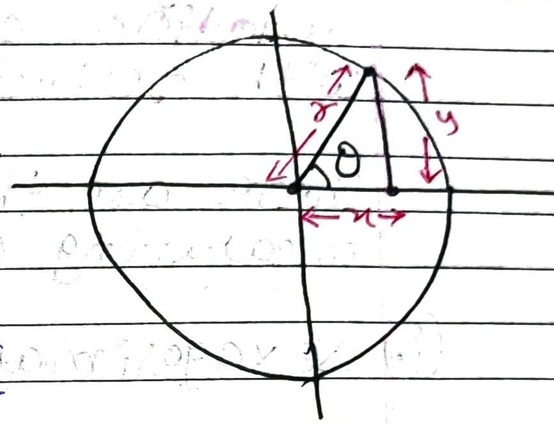
Concept 1: Trigonometric Ratios for any angle

छह trigonometric Ratios होते हैं।

☆ $\sin(\theta) = \frac{y}{r}$ $\quad \cot(\theta) = \frac{x}{y}$

$\cos(\theta) = \frac{x}{r}$ $\quad \sec(\theta) = \frac{r}{x}$

$\tan\theta = \frac{y}{x}$ $\quad \csc(\theta) = \frac{r}{y}$



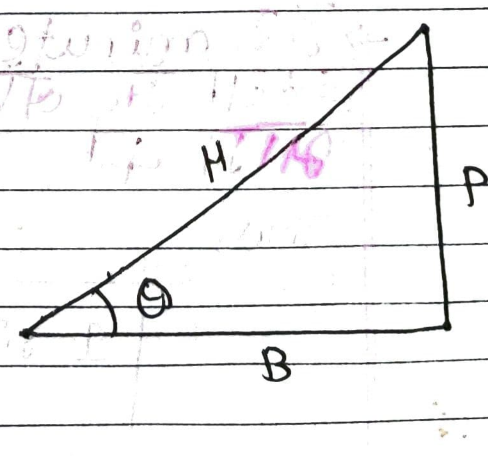
विशेषता: इस concept से हम किसी भी दिए गए angle का trigo. ratio निकाल सकते हैं। → यह ग्यारहवी का concept है।

☆ कक्षा दसवी में भी हमने Trigonometric Ratios पढ़ा था लेकिन वो सिर्फ acute angles के लिए applicable था।

$\sin\theta = \frac{P}{H}$ $\quad \cot\theta = \frac{B}{P}$

$\cos\theta = \frac{B}{H}$ $\quad \operatorname{cosec}\theta = \frac{H}{P}$

$\tan\theta = \frac{P}{B}$ $\quad \sec\theta = \frac{H}{B}$



Concept 2: Measurement of Angle

An angle is the amount of rotation of a revolving line with respect to a fixed line.

There are three system of measuring an angle :—

1) Sexagesimal System :—

→ इस व्यवस्था में एक right angle को 90 बराबर भागों में बाँटा गया है और एक भाग को कहा "degrees".

→ 1° symbol use करते हैं one degree को denote करने के लिए।

→ हर degree को further 60 बराबर हिस्सों में बाँटा गया और एक भाग को कहा "minutes".

→ हर minute को further 60 बराबर हिस्सों में बाँटा गया और एक भाग को कहा "seconds".

Thus,

$$\boxed{\begin{aligned} 1 \text{ right angle} &= 90 \text{ degrees } (=90^\circ) \\ 1^\circ &= 60 \text{ minutes } (60') \\ 1' &= 60 \text{ seconds } (60'') \end{aligned}}$$

ii) Centesimal System :-

→ इस व्यवस्था मे एक right angle को 100 बराबर भागो मे बाँटा गया और एक भाग को कहा "grades".

→ हर grade is further subdivided into 100 भागो मे बाँटा गया और एक भाग को कहा "minutes".

→ हर minute is further subdivided into 100 भागो मे बाँटा गया और एक भाग को कहा "seconds".

→ Symbols :-

1^g denotes a grade
$1'$ denotes a minute
$1''$ denotes a second

Thus,

$$\boxed{\begin{aligned} 1 \text{ right angle} &= 100 \text{ grades } (= 100^g) \\ 1 \text{ grade} &= 100 \text{ minutes } (= 100') \\ 1 \text{ minute} &= 100 \text{ seconds } (= 100'') \end{aligned}}$$

White Page

Date
Page

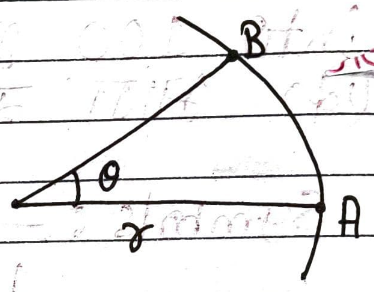
(iii) Circular System :-

→ इस व्यवस्था में unit of measurement is radion.

→ **Radians :**

One radian, written as 1^c, is the measure of an angle subtended at the centre of circle by an arc of length equal to radius of the circle.

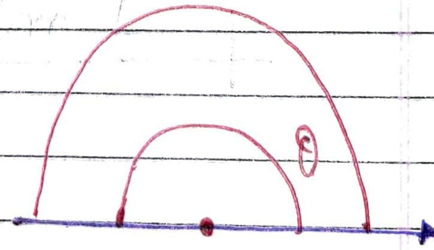
$$\theta(\text{in radians}) = \frac{\text{arc } AB}{r}$$



Relation b/w degree and radians :-

Let θ be the angle subtended at centre by a semicircle $= 180^\circ$

From figure, $\theta = 180^\circ$



In radians :

$$\theta = \frac{\text{arc}}{\text{radius}} = \frac{\pi \cdot r}{r} = \pi$$

$\Rightarrow 180^\circ$ degrees $= \pi$ radian

Remember following:-

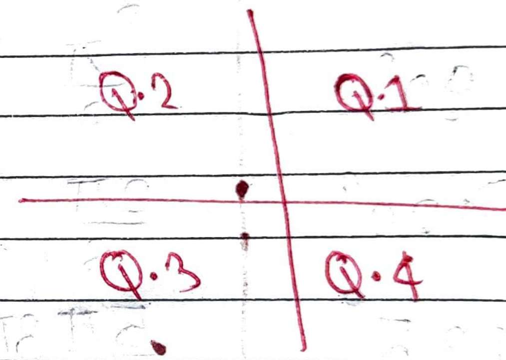
Degree	Radion	Degree	Radion
30°	$\frac{\pi}{6}$	210°	$\frac{7\pi}{6}$
45°	$\frac{\pi}{4}$	225°	$\frac{5\cdot\pi}{4}$
60°	$\frac{\pi}{3}$	240°	$\frac{4\cdot\pi}{3}$
90°	$\frac{\pi}{2}$	270°	$\frac{3\cdot\pi}{2}$
120°	$\frac{2\pi}{3}$	300°	$\frac{5\cdot\pi}{3}$
135°	$\cancel{\frac{5\pi}{6}}$ $\frac{3\pi}{4}$	315°	$\frac{7\cdot\pi}{4}$
150°	$\frac{5\pi}{6}$	330°	$\frac{11\cdot\pi}{6}$
180°	π	360°	2π

(P.T.O.)

Concept 3: Measuring of angle in various Quadrants

☆ Perpendicular lines XOX' and YOY' divide the plane in चार भागों में।

Quadrant 1:
First Quadrant
(all points have +X and +Y)

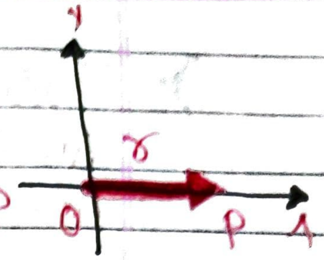


Quadrant 2:
Second Quadrant
(all points have -X and +Y)

Quadrant 3:
Third Quadrant
(all points have -X and -Y)

Quadrant 4:
Fourth Quadrant
(all points have +X and -Y)

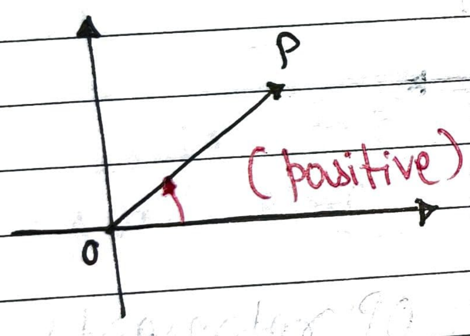
Every angle is represented by one position of a revolving ray OP of length 'r'. The starting position for ray OP is taken along +x axis.

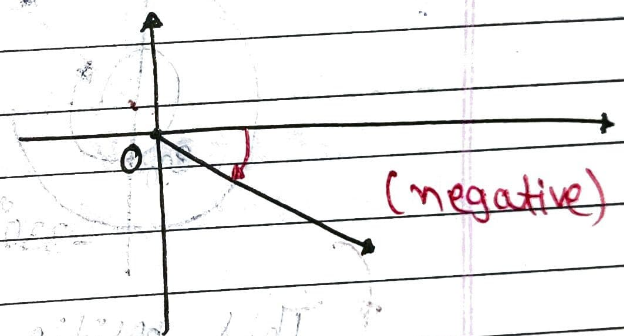


If an angle α (alpha) is positive, OP rotates through angle α in anticlockwise direction

While,

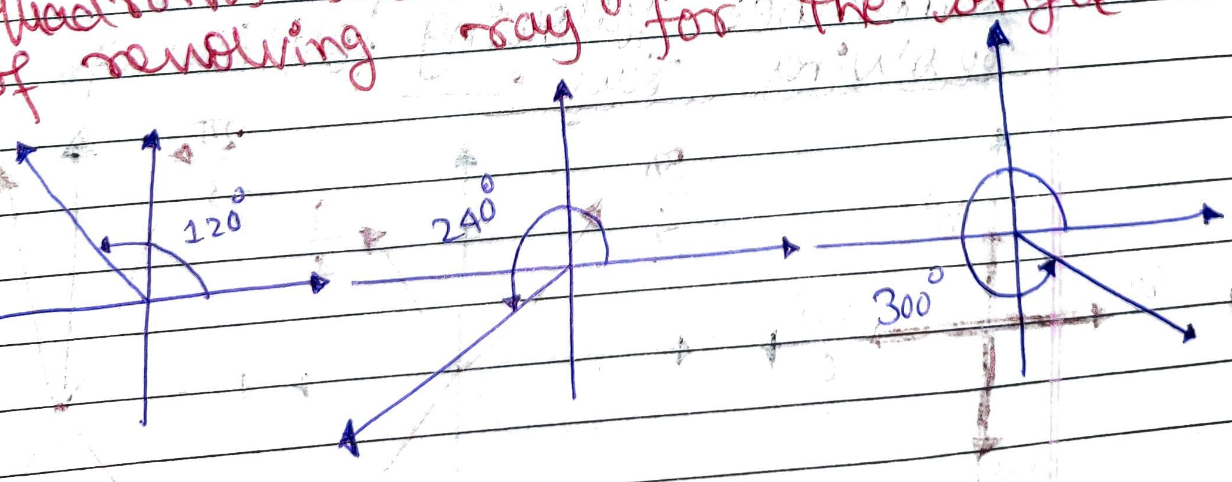
If an angle α is negative, OP rotates through angle α in clockwise direction.





An angle can lie in any of four quadrants according to the position of revolving ray for the angle.

ex:



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White Page

Date
Page

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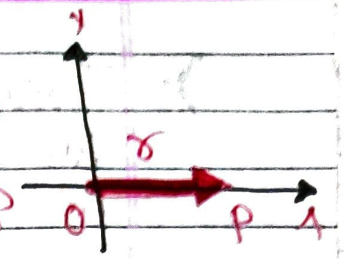

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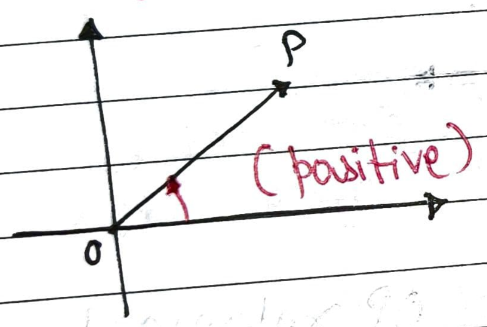

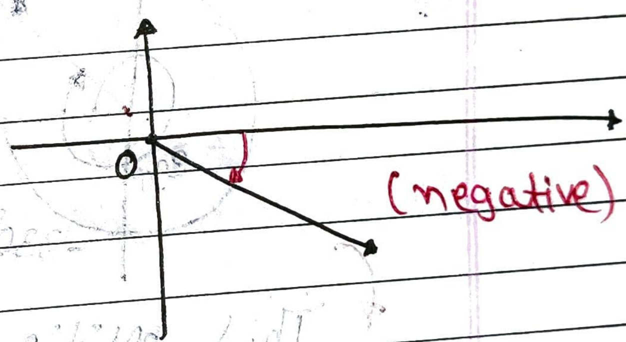

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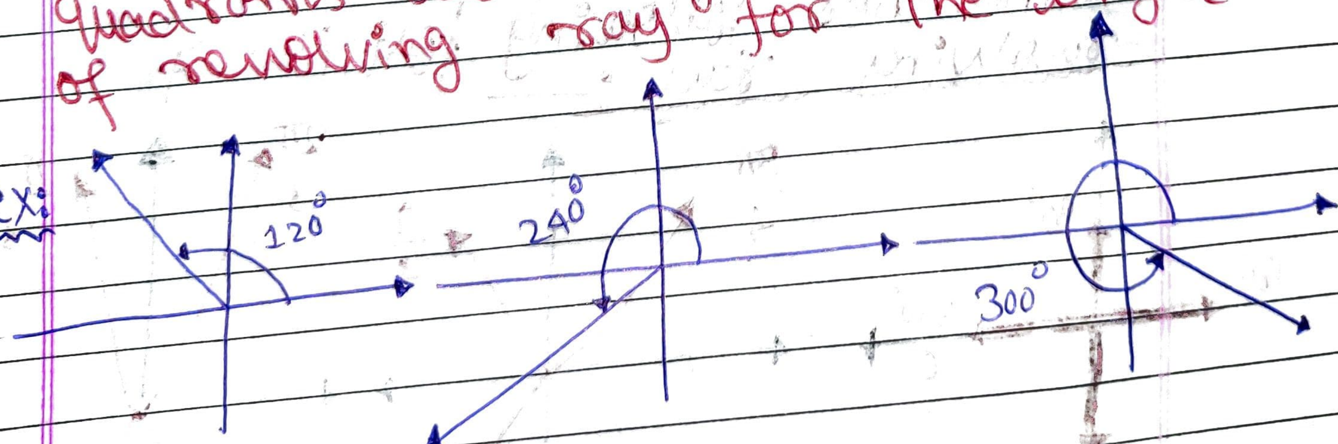

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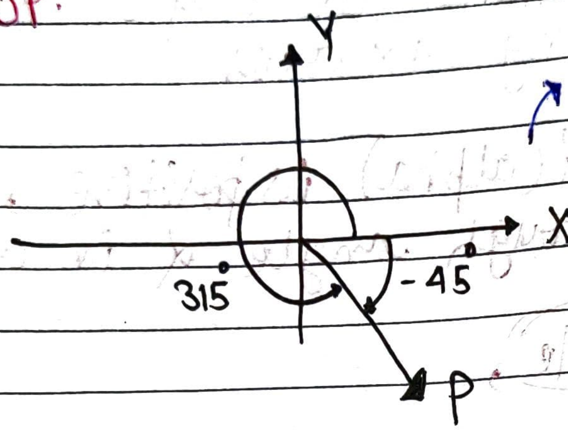
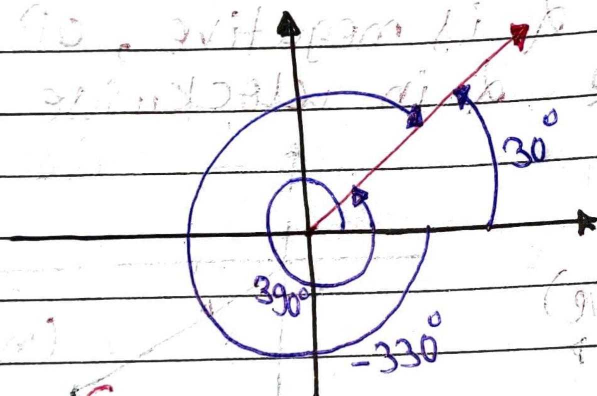
➤ Two or more angles may correspond to same position of revolving ray OP.

ex:

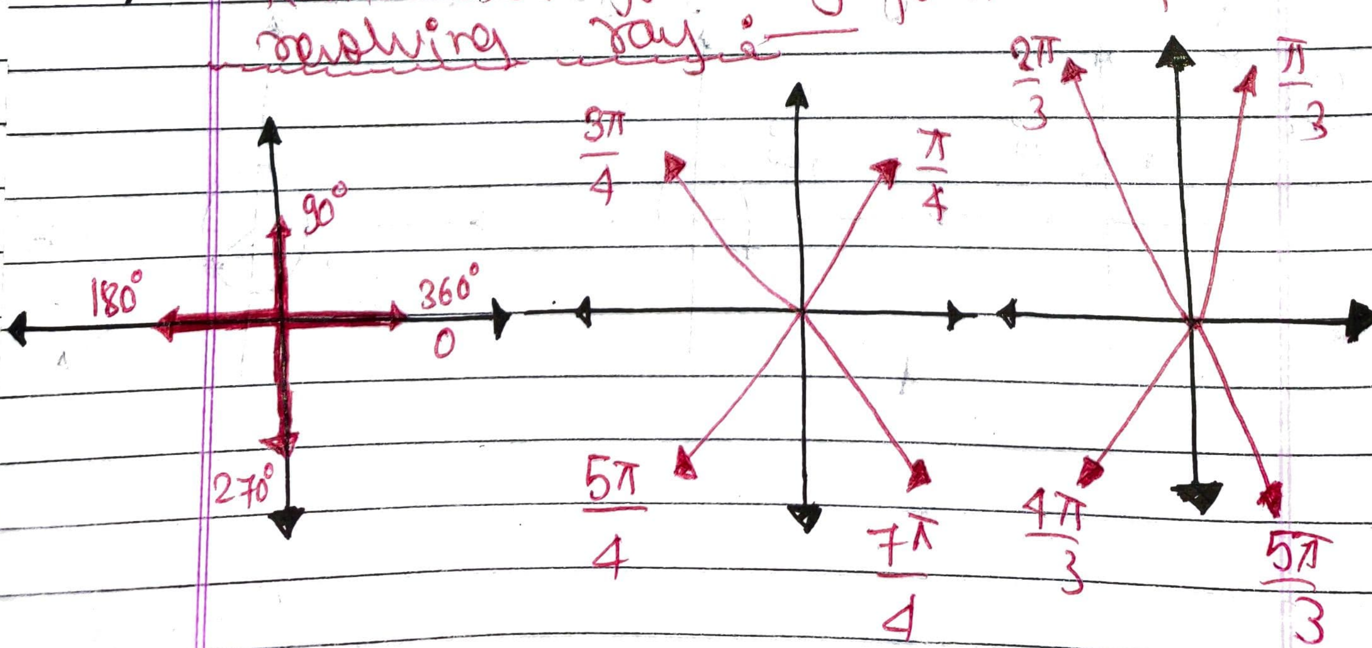


The position of OP represents -45° and $+315^\circ$



This position of OP represents $+30^\circ$, $+390^\circ$ and -330°

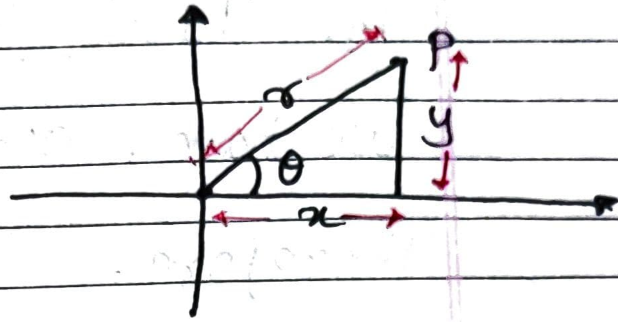
➤ Remember following position of revolving ray :-



Concept : 4

Signs of Trigonometrical Functions

In First quadrant



We have

$x > 0$, $y > 0$

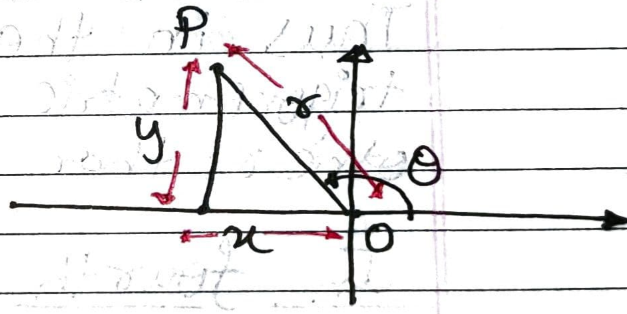
Therefore,

$\sin\theta = \frac{y}{r} > 0$, $\tan\theta = \frac{y}{x} > 0$, $\sec\theta = \frac{r}{x} > 0$

$\cos\theta = \frac{r}{y} > 0$, $\cot\theta = \frac{x}{y} > 0$, $\operatorname{cosec}\theta = \frac{r}{y} > 0$

Thus, in the first quadrant all trigonometric functions are positive.

In Second quadrant



We have $x < 0$, $y > 0$

Therefore,

$\sin\theta = \frac{y}{r} > 0$

$\cot\theta = \frac{x}{y} < 0$

$\cos\theta = \frac{x}{r} < 0$

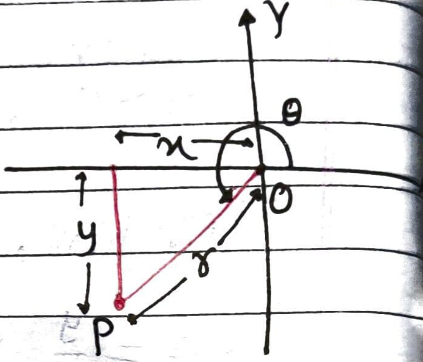
$\sec\theta = \frac{r}{x} < 0$

$\tan\theta = \frac{y}{x} < 0$

$\operatorname{cosec}\theta = \frac{r}{y} > 0$

Thus in the second quadrant all trigonometric function are negative other than sine and cosecant.

In third Quadrant



We have, $x<0$, $y<0$

Therefore,

$\sin\theta = \frac{y}{r} < 0$

$\cot\theta = \frac{x}{r} < 0$

$\cos\theta = \frac{x}{r} < 0$

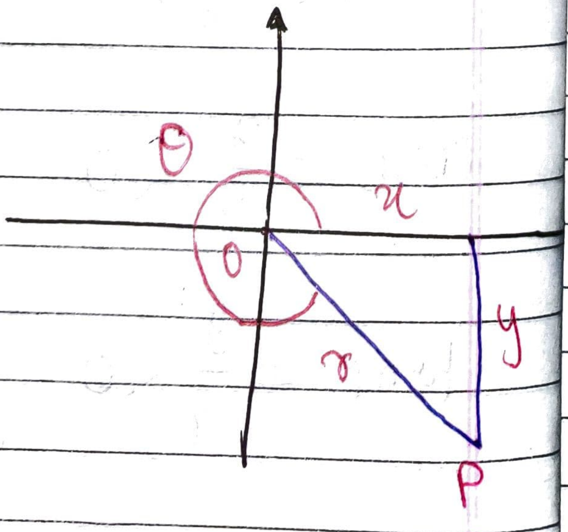
$\sec\theta = \frac{r}{x} < 0$

$\tan\theta = \frac{y}{x} > 0$

$\operatorname{cosec}\theta = \frac{r}{y} > 0$

Thus in the third quadrant all trigonometric function are negative other than tangent & cotangent.

In fourth Quadrant



Date
Page

we have $x > 0$, $y < 0$

$\sin\theta = \frac{y}{r} < 0$ $\quad$ $\operatorname{cosec}\theta = \frac{r}{y} < 0$

$\cos\theta = \frac{x}{r} > 0$ $\quad$ $\sec\theta = \frac{r}{x} > 0$

$\tan\theta = \frac{y}{x} < 0$ $\quad$ $\cot\theta = \frac{x}{y} < 0$

Thus in the fourth quadrant all trigonometric functions are negative other than cosine and secant.

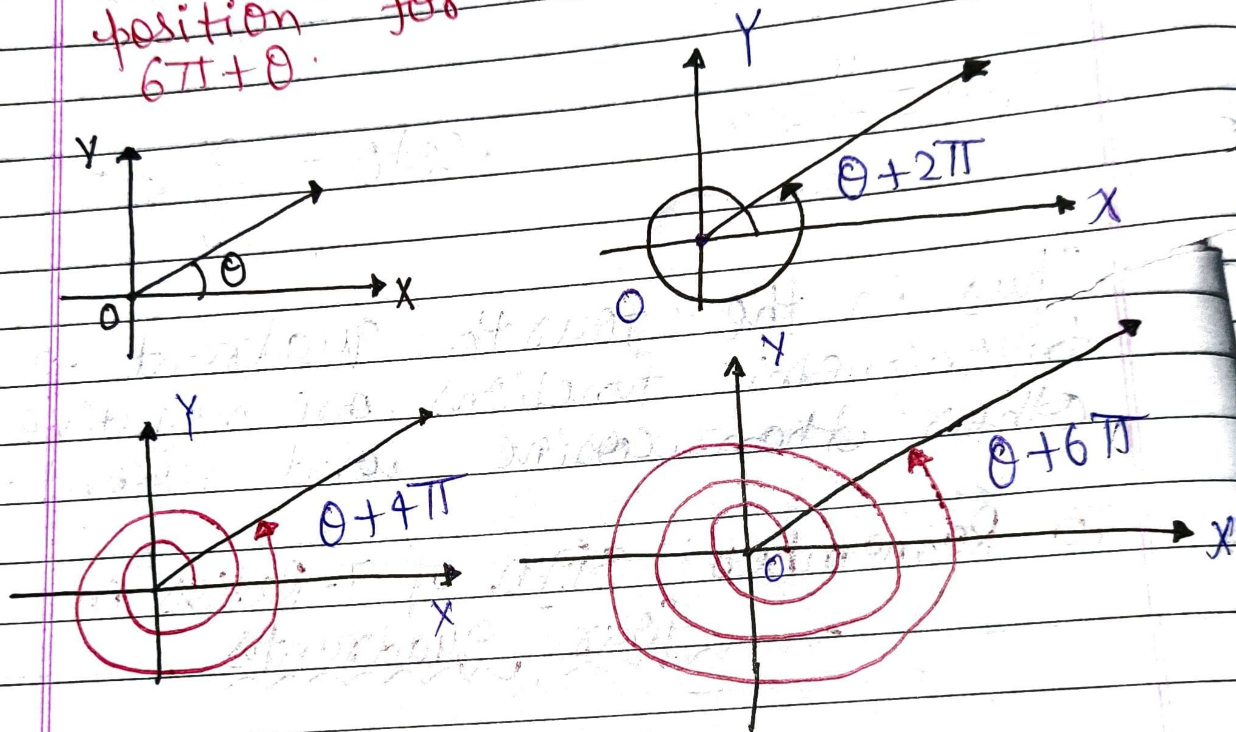
To Remember sign of T-Ratios in four quadrants

II Quadrant Sine, Cosecant	I Quadrant All positive
III Quadrant tan, Cot	IV Quadrant Cos, sec

Concept 5: Ratios of Allied angles

T-ratios for $2n\pi + \theta$:

Revolving ray assumes the same position for θ, $2\pi+\theta$, $4\pi+\theta$, $6\pi+\theta$.



$$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta$$

$$\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$$

$$\tan\left(\frac{\pi}{2}-\theta\right) = \cot\theta$$

$$\cot\left(\frac{\pi}{2}-\theta\right) = \tan\theta$$

$$\sec\left(\frac{\pi}{2}-\theta\right) = \operatorname{cosec}\theta$$

$$\operatorname{cosec}\left(\frac{\pi}{2}-\theta\right) = \sec\theta$$

White Page

Date ______

Page ______

$$\sin\left(\frac{\pi}{2}+\theta\right) = \cos\theta$$

$$\cos\left(\frac{\pi}{2}+\theta\right) = -\sin\theta$$

$$\tan\left(\frac{\pi}{2}+\theta\right) = -\cot\theta$$

$$\cot\left(\frac{\pi}{2}+\theta\right) = -\tan\theta$$

$$\sec\left(\frac{\pi}{2}+\theta\right) = -\operatorname{cosec}\theta$$

$$\operatorname{cosec}\left(\frac{\pi}{2}+\theta\right) = \sec\theta$$

$$\sin(\pi-\theta) = \sin\theta$$
$$\cos(\pi-\theta) = -\cos\theta$$
$$\tan(\pi-\theta) = -\tan\theta$$
$$\cot(\pi-\theta) = -\cot\theta$$
$$\sec(\pi-\theta) = -\sec\theta$$
$$\operatorname{cosec}(\pi-\theta) = \operatorname{cosec}\theta$$

$$\sin(\pi+\theta) = -\sin\theta$$
$$\cos(\pi+\theta) = -\cos\theta$$
$$\tan(\pi+\theta) = \tan\theta$$
$$\cot(\pi+\theta) = \cot\theta$$
$$\sec(\pi+\theta) = -\sec\theta$$
$$\operatorname{cosec}(\pi+\theta) = -\operatorname{cosec}\theta$$

$$\sin\left(\frac{3\pi}{2}-\theta\right) = -\cos\theta$$

$$\cos\left(\frac{3\pi}{2}-\theta\right) = -\sin\theta$$

$$\tan\left(\frac{3\pi}{2}-\theta\right) = \cot\theta$$

$$\cot\left(\frac{3\pi}{2}-\theta\right) = \tan\theta$$

$$\sec\left(\frac{3\pi}{2}-\theta\right) = -\operatorname{cosec}\theta$$

$$\operatorname{cosec}\left(\frac{3\pi}{2}-\theta\right) = -\sec\theta$$

$$\sin\left(\frac{3\pi}{2}+\theta\right) = -\cos\theta$$

$$\cos\left(\frac{3\pi}{2}+\theta\right) = \sin\theta$$

$$\tan\left(\frac{3\pi}{2}+\theta\right) = -\cot\theta$$

$$\cot\left(\frac{3\pi}{2}+\theta\right) = -\tan\theta$$

$$\sec\left(\frac{3\pi}{2}+\theta\right) = \operatorname{cosec}\theta$$

$$\operatorname{cosec}\left(\frac{3\pi}{2}+\theta\right) = -\sec\theta$$

Trigonometric Ratios for sum & difference of angles

* $\sin(A+B) = \sin(A)\cdot\cos(B) + \cos(A)\cdot\sin(B)$

* $\sin(A-B) = \sin(A)\cdot\cos(B) - \cos(A)\cdot\sin(B)$

* $\cos(A+B) = \cos(A)\cdot\cos(B) - \sin(A)\cdot\sin(B)$

* $\cos(A-B) = \cos(A)\cdot\cos(B) + \sin(A)\cdot\sin(B)$

* $\tan(A+B) = \dfrac{\tan(A) + \tan(B)}{1 - \tan(A)\cdot\tan(B)}$

* $\tan(A-B) = \dfrac{\tan(A) - \tan(B)}{1 + \tan(A)\cdot\tan(B)}$

* $\cot(A+B) = \dfrac{\cot(A)\cdot\cot(B) - 1}{\cot(A) + \cot(B)}$

* $\cot(A-B) = \dfrac{\cot(A)\cdot\cot(B) + 1}{\cot(B) - \cot(A)}$

White Page

Date

Page

$$\tan(A+B+C) = \frac{\tan(A) + \tan(B) + \tan(C) - \tan(A)\cdot\tan(B)\cdot\tan(C)}{1 - \tan(A)\cdot\tan(B) - \tan(B)\cdot\tan(C) - \tan(C)\tan(A)}$$

$$\cot(A+B+C) = \frac{\cot(A) + \cot(B) + \cot(C) - \cot(A)\cdot\cot(B)\cdot\cot(C)}{1 - \cot(A)\cdot\cot(B) - \cot(B)\cdot\cot(C) - \cot(C)\cdot\cot(A)}$$

$$\sin(A+B)\cdot\sin(A-B) = \sin^2(A) - \sin^2(B)$$
$$= \cos^2(B) - \cos^2(A)$$

$$\cos(A+B)\cdot\cos(A-B) = \cos^2(A) - \sin^2(B)$$
$$= \cos^2(B) - \sin^2(A)$$

$$\sin(A+B+C) = \sin(A)\cdot\cos(B)\cdot\cos(C) + \cos(A)\cdot\sin(B)\cdot\cos(C) + \cos(A)\cdot\cos(B)\cdot\sin(C) - \sin(A)\cdot\sin(B)\cdot\sin(C)$$

$$\cos(A+B+C) = \cos(A)\cdot\cos(B)\cdot\cos(C) - \sin(A)\cdot\sin(B)\cdot\cos(C) - \sin(A)\cdot\cos(B)\cdot\sin(C) - \cos(A)\cdot\sin(B)\cdot\sin(C).$$

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White Page

Date

Page

Concept 9: Trigonometric Ratios of multiple and sub-multiple angles.

(i) $\sin(2A) = 2 \cdot \sin(A) \cdot \cos(A)$

(ii) $\cos(2A) = \cos^2(A) - \sin^2(A)$

(iii) $\cos(2A) = 2 \cdot \cos^2(A) - 1$

or, $1 + \cos(2A) = 2 \cdot \cos^2(A)$

(iv) $\cos(2A) = 1 - 2\sin^2(A)$

or, $1 - \cos(2A) = 2 \cdot \sin^2(A)$

(v) $\tan(2A) = \dfrac{2 \cdot \tan(A)}{1 - \tan^2(A)}$

(vi) $\sin(2A) = \dfrac{2\tan(A)}{1 + \tan^2(A)}$

(vii) $\cos(2A) = \dfrac{1 - \tan^2(A)}{1 + \tan^2(A)}$

(viii) $\sin(3A) = 3\sin(A) - 4 \cdot \sin^3(A)$

(ix) $\cos(3A) = 4 \cdot \cos^3(A) - 3 \cdot \cos(A)$

(x) $\tan(3A) = \dfrac{3\tan(A) - \tan^3(A)}{1 - 3 \cdot \tan^2(A)}$

White Page

Date

Page

Concept 10: Transformation formulae

★ Expressing Product of Trigonometric Functions as Sum or Difference

(i) $2\cdot \sin(A)\cdot \cos(B) = \sin(A+B) + \sin(A-B)$

(ii) $2\cdot \cos(A)\cdot \sin(B) = \sin(A+B) - \sin(A-B)$

(iii) $2\cdot \cos(A)\cdot \cos(B) = \cos(A+B) + \cos(A-B)$

(iv) $2\cdot \sin(A)\cdot \sin(B) = \cos(A-B) - \cos(A+B)$

★ Expressing Sum or Difference of two sines or two cosines as a product

$$\sin(C) + \sin(D) = 2\cdot \sin\left(\frac{C+D}{2}\right)\cdot \cos\left(\frac{C-D}{2}\right)$$

$$\sin(C) - \sin(D) = 2\cdot \sin\left(\frac{C-D}{2}\right)\cdot \cos\left(\frac{C+D}{2}\right)$$

$$\cos(C) + \cos(D) = 2\cdot \cos\left(\frac{C+D}{2}\right)\cdot \cos\left(\frac{C-D}{2}\right)$$

$$\cos(C) - \cos(D) = -2\cdot \sin\left(\frac{C+D}{2}\right)\cdot \sin\left(\frac{C-D}{2}\right)$$

Practice Problems

Q. Find the value of :—

(i.) $\sin\left(\frac{25\pi}{3}\right) = \sin\left(8\pi + \frac{\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$\left[\because \sin(2n\pi + \theta) = \sin(\theta)\right]$

(ii.) $\cos\left(\frac{41\pi}{4}\right) = \cos\left(10\pi + \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

(iii.) $\tan\left(-\frac{16\pi}{3}\right) = \tan\left(-5\pi - \frac{\pi}{3}\right) = -\tan\left(5\pi + \frac{\pi}{3}\right)$

$= -\tan\left(\frac{\pi}{3}\right) = -\sqrt{3}$

(iv.) $\cot\left(\frac{29\pi}{4}\right) = \cot\left(7\pi + \frac{\pi}{4}\right) = \cot\left(\frac{\pi}{4}\right) = 1$

(V.) $\sec\left(-\frac{19\pi}{3}\right) = \sec\left(-6\pi - \frac{\pi}{3}\right) = \sec\left(6\pi + \frac{\pi}{3}\right)$

$= \sec\left(\frac{\pi}{3}\right) = 2.$

(vi.) $\operatorname{cosec}\left(-\frac{33\pi}{4}\right) = -\operatorname{cosec}\left(\frac{33\pi}{4}\right)$

$= -\operatorname{cosec}\left(8\pi + \frac{\pi}{4}\right) = -\operatorname{cosec}\left(\frac{\pi}{4}\right)$

$= -\sqrt{2}$

Q. Find the value of :—

(i.) $\sin(765^\circ)$

Ans: $\because 180^\circ = \pi^c \Rightarrow 765^\circ = \left(\frac{\pi}{180^\circ} \times 765\right)^c$

$= \left(\frac{17\pi}{4}\right)^c$

$\therefore \sin(765^\circ)$

$= \sin\left(4\pi + \frac{\pi}{4}\right)$

$= \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

(ii.) $\text{Cosec}(-1110^\circ)$

Ans: $\because 180^\circ = \pi^c$

$\Rightarrow 1110^\circ = \left(\frac{\pi}{180} \times 1110\right)^c = \left(\frac{37\pi}{6}\right)^c$

$\therefore \text{Cosec}(-1110^\circ) = -\text{cosec}(1110^\circ)$

$= -\text{cosec}\left(\frac{37\pi}{6}\right)$

$= -\text{cosec}\left(6\pi + \frac{\pi}{6}\right)$

$= -\text{cosec}\left(\frac{\pi}{6}\right)$

$= -2$

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White Page

Date ______

Page ______

(iii) $\cot(-600^\circ)$

Ans: $\because 180^\circ = \pi^c$

$$\Rightarrow 600^\circ = \left(\frac{\pi}{180} \times 600\right)^c = \left(\frac{10\pi}{3}\right)^c$$

$$\therefore \cot(-600^\circ) = -\cot(600^\circ)$$

$$= -\cot\left(\frac{10\pi}{3}\right)$$

$$= -\cot\left(3\pi + \frac{\pi}{3}\right) = -\cot\left(\frac{\pi}{3}\right)$$

$$= -\frac{1}{\sqrt{3}}$$

Q. Find the value of :—

(i.) $\cos(15\pi) = \cos(14\pi + \pi)$
$= \cos(\pi) = -1$

(ii.) $\sin(16\pi) = \sin(16\pi + 0)$
$= \sin 0^\circ = 0$

(iii.) $\cos(-\pi) = \cos(\pi) = -1$

(iv.) $\sin(5\pi) = \sin(4\pi + \pi)$
$= \sin(\pi) = 0$

(v.) $\tan\left(\frac{5\pi}{4}\right) = \quad \because \frac{5\pi}{4} = 225^\circ \rightarrow 180^\circ + 45^\circ$

So, $= \tan\left(\pi + \frac{\pi}{4}\right)$

$= \tan\left(\frac{\pi}{4}\right) = 1$

Page

Q. Find the value of :—

(i.) $\sin\left(\frac{31\pi}{3}\right) = \sin\left(10\pi + \frac{\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right) = \cancel{\frac{\sqrt{3}}{2}}$

$= \frac{\sqrt{3}}{2}$

(ii.) $\cos\left(\frac{17\pi}{2}\right) = \cos\left(8\pi + \frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2}\right)$

$= 0$

(iii.) $\tan\left(-\frac{25\pi}{3}\right) = -\tan\left(\frac{25\pi}{3}\right)$

$= -\tan\left(8\pi + \frac{\pi}{3}\right) = -\tan\left(\frac{\pi}{3}\right)$

$= -\sqrt{3}$

(iv.) $\cot\left(\frac{13\pi}{4}\right) = \cot\left(3\pi + \frac{\pi}{4}\right)$

$= \cot\left(\frac{\pi}{4}\right) = 1$

(v.) $\sec\left(-\frac{25\pi}{3}\right) = \sec\left(\frac{25\pi}{3}\right)$

$= \sec\left(8\pi + \frac{\pi}{3}\right) = \sec\left(\frac{\pi}{3}\right) = 2$

(vi.) $\operatorname{cosec}\left(-\frac{41\pi}{4}\right) = -\operatorname{cosec}\left(\frac{41\pi}{4}\right)$

$= -\operatorname{cosec}\left(10\pi + \frac{\pi}{4}\right) = -\operatorname{cosec}\left(\frac{\pi}{4}\right)$

$= -\sqrt{2}$

Q. Find the value of :-

(i) $\sin(405^\circ) =$

$\because 180^\circ = \pi^c$

$\Rightarrow 405^\circ = \left(\frac{\pi}{\cancel{180^\circ}_{\cancel{36}\,4}} \times \cancel{405}^{\cancel{81}\,9}\right)^c$

$\Rightarrow \frac{9\pi}{4} = 2\pi + \frac{\pi}{4}$

$\Rightarrow \sin\left(2\pi + \frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

(ii) $\sec(-1470^\circ) =$

$\because 180^\circ = \pi^c$

$\Rightarrow -1470^\circ = -\left(\frac{\pi}{\cancel{180}_{6}} \times \cancel{1470}^{49}\right)^c$

$\Rightarrow \sec\left(-\frac{49\cdot\pi}{6}\right) = \sec\left(\frac{49\cdot\pi}{6}\right)$

$= \sec\left(8\pi + \frac{\pi}{6}\right) = \sec\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}}$

(iii) $\tan(-300^\circ) =$

$\because 180^\circ = \pi^c$

$\Rightarrow -300^\circ = -\left(\frac{\pi}{\cancel{180}_{\cancel{6}\,3}} \times \cancel{300}^{\cancel{10}\,5}\right)^c = -\frac{5\cdot\pi}{3}$

$\Rightarrow \tan\left(-\frac{5\cdot\pi}{3}\right) = -\tan\left(\frac{5\pi}{3}\right) = -\tan\left(2\pi - \frac{\pi}{3}\right)$

$\Rightarrow +\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$

(iv) $\cot(585^\circ)$

Ans: $\because 180^\circ = \pi^c$

$\Rightarrow 585^\circ = \left(\frac{\pi}{\cancel{180}_{\cancel{36}\ \cancel{12}\ 4}} \times \cancel{585}^{\cancel{117}\ \cancel{39}\ 13}\right)^c = \frac{13\pi}{4}$

$\Rightarrow \cot\left(\frac{13\pi}{4}\right) = \cot\left(3\pi + \frac{\pi}{4}\right) = \cancel{\cot\left(\frac{\pi}{4}\right)}$

$= \cot\left(\frac{\pi}{4}\right) = 1.$

(v) $\operatorname{cosec}(-750^\circ)$

Ans: $\because 180^\circ = \pi^c$

$\Rightarrow -750^\circ = -\left(\frac{\pi}{18\cancel{0}} \times 75\cancel{0}\right)^c \cancel{= -\left(\frac{72\pi}{\right.}}$

$\Rightarrow -\operatorname{cosec}\left(\frac{75\pi}{18}\right)^c = -\operatorname{cosec}\left(4\pi + \frac{3\pi}{18}\right)$

$\Rightarrow -\operatorname{cosec}\left(\frac{\cancel{3}\pi}{\cancel{18}_{6}}\right) = -\operatorname{cosec}\left(\frac{\pi}{6}\right)$

$= -2$

(vi) $\cos(-2220^\circ)$

Ans: $\Rightarrow \cos\left(\frac{\pi}{\cancel{180}_{\cancel{9}\ 3}} \times \cancel{2220}^{\cancel{111}\ 37}\right)^c = \cos\left(\frac{37\pi}{3}\right)^c$

$= \cos\left(12\pi + \frac{\pi}{3}\right)^c$

$= \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$.

Q. Find the value of $\sin^2\left(\frac{\pi}{6}\right) + \cos^2\left(\frac{\pi}{3}\right) - \tan^2\left(\frac{\pi}{4}\right)$.

Ans: $= \sin^2\left(\frac{\pi}{6}\right) + \cos^2\left(\frac{\pi}{3}\right) - \tan^2\left(\frac{\pi}{4}\right)$

$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 - (1)^2 = \frac{\cancel{2}}{\cancel{4}_2} - 1 = -\frac{1}{2}$.

Q. Find the value of $\tan^2\left(\frac{\pi}{3}\right) + 2\cdot\cos^2\left(\frac{\pi}{4}\right) + 3\cdot\sec^2\left(\frac{\pi}{6}\right) + 4\cdot\cos^2\left(\frac{\pi}{2}\right)$

Ans: $\cancel{= (\sqrt{3})^2 + 2\cdot\left(\frac{1}{\sqrt{2}}\right)^2 + 3\cdot\left(\frac{\sqrt{3}}{2}\right)^2 + 4\cdot 0}$

$\cancel{= 3 + 1 + \frac{3}{4} + 0}$

$\cancel{= 4 + \frac{3\times 3}{4}}$

$= (\sqrt{3})^2 + 2\cdot\left(\frac{1}{\sqrt{2}}\right)^2 + 3\cdot\left(\frac{2}{\sqrt{3}}\right)^2 + 4\cdot(0)^2$

$= 3 + 1 + 4 + 0 = 8$

Q. find the value of :-

$4\cdot\sin\left(\frac{\pi}{6}\right)\cdot\sin^2\left(\frac{\pi}{3}\right) + 3\cdot\cos\left(\frac{\pi}{3}\right)\cdot\tan\left(\frac{\pi}{4}\right) + \operatorname{cosec}^2\left(\frac{\pi}{2}\right)$

Ans:

$= \cancel{4}^{2} \times \frac{1}{\cancel{2}} \times \frac{3}{4} + 3 \times \frac{1}{2} \times 1 + (1)^2$

$= \cancel{2} \times \frac{3}{\cancel{4}_2} + \frac{3}{2} + 1$

$= \frac{3}{2} + \frac{3}{2} + 1 = \frac{\cancel{8}^{4}}{\cancel{2}} = 4$

White Page

Date

Page

Q. Find the value of :-

(i.) $\cos(480^\circ)$

Ans: $= \cos(360^\circ + 120^\circ)$

$= \cos(120^\circ)$

$= \cos(180 - 60^\circ)$

$= -\cos(60^\circ)$

$= -\frac{1}{2}$.

(ii.) $\sin(1230^\circ)$

Ans: $= \sin(3 \times 360^\circ + 150^\circ)$

$= \sin(150^\circ)$

$= \sin(180^\circ - 30^\circ)$

$= \sin(30^\circ) = \frac{1}{2}$.

(iii.) $\cot(-135^\circ)$

Ans: $= -\cot(135^\circ)$

$= -\cot(90 + 45^\circ)$

$= \tan 45^\circ = 1$.

(iv.) $\operatorname{cosec}(-1410^\circ)$

Ans: $= -\operatorname{cosec}(1410^\circ)$

$= -\operatorname{cosec}(4 \times 360^\circ - 30^\circ)$

$= \operatorname{cosec}(30^\circ)$

$= 2$

(v) $\cos(-870^\circ)$

$= \cos(870^\circ)$

$= \cos(2 \times 360^\circ + 150^\circ)$

$= \cos(150^\circ)$

$= \cos(180^\circ - 30^\circ)$

$= -\cos(30^\circ) = -\frac{\sqrt{3}}{2}.$

(vi) $\tan(330^\circ)$

$= \tan(360^\circ - 30^\circ)$

$= -\tan 30^\circ = -\frac{1}{\sqrt{3}}.$

Q. Find the values of all trigonometric functions of :-

(i) 120° :-

$\sin(120^\circ) = \sin(180^\circ - 60^\circ) = \sin(60^\circ) = \frac{\sqrt{3}}{2}$

$\cos(120^\circ) = \cos(180^\circ - 60^\circ) = -\cos(60^\circ) = -\frac{1}{2}$

$\tan(120^\circ) = \frac{\sin 120^\circ}{\cos 120^\circ} = \frac{\sqrt{3}}{2} \times (-2) = -\sqrt{3}$

$\operatorname{cosec}(120^\circ) = \frac{1}{\sin(120^\circ)} = \frac{2}{\sqrt{3}}$

$\sec(120^\circ) = \frac{1}{\cos(120^\circ)} = -2$

$\cot(120^\circ) = \frac{1}{\tan(120^\circ)} = -\frac{1}{\sqrt{3}}$

(ii) 150°

$\sin(150^\circ) = \sin(180^\circ - 30^\circ) = \sin(30^\circ) = \frac{1}{2}$

$\cos(150^\circ) = \cos(180^\circ - 30^\circ) = -\cos(30^\circ) = -\frac{\sqrt{3}}{2}$

$\tan(150^\circ) = \frac{\sin(150^\circ)}{\cos(150^\circ)} = \frac{1}{2} \times \frac{2}{-\sqrt{3}} = \frac{-1}{\sqrt{3}}$

$\cot(150^\circ) = \frac{1}{\tan(150^\circ)} = -\sqrt{3}$

$\sec(150^\circ) = \frac{1}{\cot(150^\circ)} = -\frac{2}{\sqrt{3}}$

$\operatorname{cosec}(150^\circ) = \frac{1}{\sin(150^\circ)} = 2$

Q Show that :-

$$\sin(105^\circ) + \cos(105^\circ) = \frac{1}{2}$$

Ans:- $L.H.S. = \sin(105^\circ) + \cos(105^\circ)$

$= \sin(60^\circ + 45^\circ) + \cos(60^\circ + 45^\circ)$

$= (\sin 60^\circ \cdot \cos 45^\circ + \cos 60^\circ \cdot \sin 45^\circ) +$
$(\cos 60^\circ \cdot \cos 45^\circ - \sin 60^\circ \cdot \sin 45^\circ)$

$= \left\{\left(\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}\right) + \left(\frac{1}{2} \times \frac{1}{\sqrt{2}}\right)\right\} +$

$\left\{\left(\frac{1}{2} \times \frac{1}{\sqrt{2}}\right) - \left(\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}\right)\right\}$

$$= \left(\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}\right) = \frac{1}{\sqrt{2}}.$$

★★ Q Calculate the value of :–

(i.) $\sin 15^\circ$:-

$$= \sin(45^\circ - 30^\circ)$$

$$= \sin 45^\circ \cdot \cos 30^\circ - \cos 45^\circ \cdot \sin 30^\circ$$

$$= \left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}} \times \frac{1}{2}\right)$$

$$= \frac{\sqrt{3} - 1}{2 \cdot \sqrt{2}}$$

(ii.) $\cos 15^\circ$:–

$$= \cos(45^\circ - 30^\circ)$$

$$= \cos 45^\circ \cdot \cos 30^\circ + \sin 45^\circ \cdot \sin 30^\circ$$

$$= \left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}} \times \frac{1}{2}\right)$$

$$= \frac{\sqrt{3} + 1}{2 \cdot \sqrt{2}}$$

(iii.) $\tan 15^\circ$:–

$$= \tan(45^\circ - 30^\circ)$$

$$= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \cdot \tan 30^\circ}$$

$$= \frac{1 - 1/\sqrt{3}}{1 + 1 \times \frac{1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

(iv) $\sin(75^\circ)$

$= \sin(45^\circ + 30^\circ)$

$= \sin 45^\circ \cdot \cos 30^\circ + \cos 45^\circ \cdot \sin 30^\circ$

$= \left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}} \times \frac{1}{2}\right)$

$= \frac{\sqrt{3}+1}{2\cdot\sqrt{2}}$

(v) $\cos(75^\circ)$

$= \cos(45^\circ + 30^\circ)$

$= \cos(45^\circ)\cdot\cos(30^\circ) - \sin(45^\circ)\cdot\sin(30^\circ)$

$= \left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}} \times \frac{1}{2}\right)$

$= \frac{\sqrt{3}-1}{2\cdot\sqrt{2}}$

(vi) $\tan(75^\circ)$

$= \tan(45^\circ + 30^\circ)$

$= \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ}$

$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

Q. Evaluate $\tan\left(\frac{13\pi}{12}\right)$

Ans: $\tan\left(\frac{13\pi}{12}\right) = \tan\left(\pi + \frac{\pi}{12}\right)$

$= \tan\left(\frac{\pi}{12}\right) = \tan\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$$= \frac{\tan\left(\frac{\pi}{3}\right) - \tan\left(\frac{\pi}{4}\right)}{1 + \tan\left(\frac{\pi}{3}\right)\tan\left(\frac{\pi}{4}\right)}$$

$$= \frac{\sqrt{3} - 1}{1 + \sqrt{3}}$$

$$= \frac{(\sqrt{3}-1)}{(\sqrt{3}+1)} \times \frac{(\sqrt{3}-1)}{(\sqrt{3}-1)} = \frac{(\sqrt{3}-1)^2}{3-1}$$

$$= \frac{3+1-2\sqrt{3}}{2} = \frac{4-2\sqrt{3}}{2} = 2-\sqrt{3}$$

Q. Prove that :—

(i) $\sin 70^\circ \cdot \cos 10^\circ - \cos 70^\circ \cdot \sin 10^\circ = \frac{\sqrt{3}}{2}$

Ans: $\because \sin\theta \cdot \cos\phi - \cos\theta \cdot \sin\phi = \sin(\theta - \phi)$

so $= \sin(70 - 10^\circ)$

$= \sin(60^\circ) = \frac{\sqrt{3}}{2}$.

(ii.) $\cos 50^\circ \cdot \cos 10^\circ - \sin 50^\circ \cdot \sin 10^\circ = \frac{1}{2}$.

Ans: $\because \cos\theta \cdot \cos\phi - \sin\theta \cdot \sin\phi = \cos(\theta+\phi)$

$= \cos(50^\circ + 10^\circ) = \cos 60^\circ = \frac{1}{2}$.

(iii.) $\cos 80^\circ \cdot \cos 20^\circ + \sin 80^\circ \cdot \sin 20^\circ = \frac{1}{2}$.

Ans: $\because \cos\theta \cdot \cos\phi + \sin\theta \cdot \sin\phi = \cos(\theta - \phi)$

$= \cos(80^\circ - 20^\circ) = \cos 60^\circ = \frac{1}{2}$.

(iv.) $\sin 36^\circ \cdot \cos 9^\circ + \cos 36^\circ \cdot \sin 9^\circ = \frac{1}{\sqrt{2}}$

Ans: $\because \sin\theta \cdot \cos\phi + \cos\theta \cdot \sin\phi = \sin(\theta+\phi)$

$= \sin(36^\circ + 9^\circ) = \sin 45^\circ = \frac{1}{\sqrt{2}}$.

Q. Prove that :-

$\sin(40^\circ+\theta) \cdot \cos(10^\circ+\theta) - \cos(40^\circ+\theta) \cdot \sin(10^\circ+\theta)$
$= 1/2$.

Ans:

L.H.S. $= \sin(40^\circ+\theta) \cdot \cos(10^\circ+\theta) - \cos(40^\circ+\theta) \cdot \sin(20^\circ+\theta)$
$= \sin[(40^\circ+\theta) - (10^\circ+\theta)]$
$= \sin(30^\circ)$
$= \frac{1}{2}$.

Q. Prove that :-

$$\cos\theta + \sin(270^\circ+\theta) - \sin(270^\circ-\theta) + \cos(180^\circ+\theta) = 0$$

Ans: $= \cos\theta + \sin[180^\circ + (90^\circ+\theta)] - \sin[180^\circ + (90^\circ - \theta)] + \cos(180^\circ+\theta)$

$= \cos\theta - \sin(90^\circ+\theta) + \sin(90^\circ-\theta) - \cos\theta$

$= \cancel{\cos\theta} - \cos\theta + \cos\theta - \cancel{\cos\theta} = 0.$

Q. Prove that :-

$$\frac{\cos(90^\circ+\theta)\cdot\sec(270^\circ+\theta)\cdot\sin(180^\circ+\theta)}{\operatorname{cosec}(-\theta)\cos(270^\circ-\theta)\cdot\tan(180^\circ+\theta)} = \cos\theta.$$

Ans: L.H.S.

$$= \frac{\cos(90^\circ+\theta)\cdot\sec[180^\circ+(90^\circ+\theta)]\cdot\sin(180^\circ+\theta)}{-\operatorname{cosec}(\theta)\cdot\cos[180^\circ+(90^\circ-\theta)]\cdot\tan(180^\circ+\theta)}$$

$$= \frac{\cos(90^\circ+\theta)\cdot\{-\sec(90^\circ+\theta)\}\{-\sin(\theta)\}}{-\operatorname{cosec}(\theta)\cdot\{-\cos(90^\circ-\theta)\}\cdot\tan(\theta)}$$

$$= \frac{(-\sin\theta)\cdot\operatorname{cosec}\theta\cdot(-\sin\theta)}{(-\operatorname{cosec}\theta)\cdot(-\sin\theta)\cdot\tan\theta}$$

$= \cos\theta$

$=$ R.H.S.

Q. Prove that :-

$$\cos\left(\frac{3\pi}{2}+\theta\right)\cdot\cos(2\pi+\theta)\left[\cot\left(\frac{3\pi}{2}-\theta\right)+\cot(2\pi+\theta)\right]=1$$

Ans:

$$=\cos\left(\frac{3\pi}{2}+\theta\right)\cdot\cos(2\pi+\theta)\left[\cot\left(\frac{3\pi}{2}-\theta\right)+\cot(2\pi+\theta)\right]$$

$$=\cos\left[\pi+\left(\frac{\pi}{2}+\theta\right)\right]\cdot\cos(2\pi+\theta)\left[\cot\left\{\pi+\left(\frac{\pi}{2}-\theta\right)\right\}+\cot(2\pi+\theta)\right]$$

$$=-\cos\left(\frac{\pi}{2}+\theta\right)\cdot\cos\theta\left[\cot\left(\frac{\pi}{2}-\theta\right)+\cot\theta\right]$$

$$=\sin\theta\cdot\cos\theta\left[\tan\theta+\cot\theta\right]$$

$$=(\sin\theta\cdot\cos\theta)\left[\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right]$$

$$=(\cancel{\sin\theta\cdot\cos\theta})\left[\frac{\sin^2\theta+\cos^2\theta}{\cancel{\sin\theta\cdot\cos\theta}}\right]=1.$$

Q. Prove that :-

$$\sin\left(\frac{7\pi}{12}\right)\cdot\cos\left(\frac{\pi}{4}\right)-\cos\left(\frac{7\pi}{12}\right)\cdot\sin\left(\frac{\pi}{4}\right)=\frac{\sqrt{3}}{2}$$

Ans:

$$\left\{\because \sin x\cdot\cos y-\cos x\cdot\sin y=\sin(x-y)\right\}$$

$$=\sin\left(\frac{7\pi}{12}-\frac{\pi}{4}\right)$$

$$=\sin\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}$$

Q. Prove that :-

$$\sin\left(\frac{\pi}{4}\right)\cdot\cos\left(\frac{\pi}{12}\right)+\cos\left(\frac{\pi}{4}\right)\cdot\sin\left(\frac{\pi}{12}\right)=\frac{\sqrt{3}}{2}$$

Ans: $\left\{\because \sin x\cdot\cos y+\cos x\cdot\sin y=\sin(x+y)\right\}$

$$=\sin\left(\frac{\pi}{4}+\frac{\pi}{12}\right)=\sin\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}$$

Q. Prove that :-

$$\cos\left(\frac{2\cdot\pi}{3}\right)\cdot\cos\left(\frac{\pi}{4}\right)-\sin\left(\frac{2\cdot\pi}{3}\right)\cdot\sin\left(\frac{\pi}{4}\right)=\frac{-(\sqrt{3}+1)}{2\cdot\sqrt{2}}$$

Ans:

$$=\cos\left(\pi-\frac{\pi}{3}\right)\cdot\cos\left(\frac{\pi}{4}\right)-\sin\left(\pi-\frac{\pi}{3}\right)\cdot\sin\left(\frac{\pi}{4}\right)$$

$$=-\cos\left(\frac{\pi}{3}\right)\cdot\cos\left(\frac{\pi}{4}\right)-\sin\left(\frac{\pi}{3}\right)\cdot\overset{\sin}{\cancel{\cos}}\left(\frac{\pi}{4}\right)$$

$$=-\left(\frac{1}{2}\times\frac{1}{\sqrt{2}}\right)-\left(\frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}}\right)$$

$$=-\left\{\frac{(\sqrt{3}+1)}{2\sqrt{2}}\right\}$$

Q. Evaluate :

(1.) $\sin\left(\frac{\pi}{12}\right)$

Ans: $=\sin\left(\frac{\pi}{4}-\frac{\pi}{6}\right)$

$$=\sin\left(\frac{\pi}{4}\right)\cdot\cos\left(\frac{\pi}{6}\right)-\cos\left(\frac{\pi}{4}\right)\cdot\sin\left(\frac{\pi}{6}\right)$$

$$=\left(\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}\right)-\left(\frac{1}{\sqrt{2}}\times\frac{1}{2}\right)$$

$$=\frac{(\sqrt{3}-1)}{2\cdot\sqrt{2}}$$

White Page
Date
Page

(ii.) $\sin\left(\frac{5\pi}{12}\right)$

Ans:

$= \sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$

$= \sin\left(\frac{\pi}{4}\right) \cdot \cos\left(\frac{\pi}{6}\right) + \cos\left(\frac{\pi}{4}\right) \cdot \sin\left(\frac{\pi}{6}\right)$

$= \left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}} \times \frac{1}{2}\right)$

$= \frac{(\sqrt{3}+1)}{2 \cdot \sqrt{2}}$

Q. Prove that :-

$$\frac{\sin(x+y)}{\sin(x-y)} = \frac{\tan x + \tan y}{\tan x - \tan y}$$

Ans:

$$\text{L.H.S.} = \frac{\sin(x+y)}{\sin(x-y)}$$

$$= \frac{\sin x \cdot \cos y + \cos x \cdot \sin y}{\sin x \cdot \cos y - \cos x \cdot \sin y}$$

$$= \frac{\frac{\sin x \cdot \cos y}{\cos x \cdot \cos y} + \frac{\cos x \cdot \sin y}{\cos x \cdot \cos y}}{\frac{\sin x \cdot \cos y}{\cos x \cdot \cos y} - \frac{\cos x \cdot \sin y}{\cos x \cdot \cos y}}$$

$$= \frac{\tan x + \tan y}{\tan x - \tan y} = R.H.S.$$

Q. Prove that :—

$$\tan(56^\circ) = \frac{\cos(11^\circ) + \sin(11^\circ)}{\cos(11^\circ) - \sin(11^\circ)}$$

Ans:

$$L.H.S. = \tan(56^\circ)$$

$$= \tan(45^\circ + 11^\circ)$$

$$= \frac{\tan 45^\circ + \tan 11^\circ}{1 - \tan 45^\circ \cdot \tan 11^\circ}$$

$$= \frac{1 + \tan 11^\circ}{1 - \tan 11^\circ}$$

$$= \frac{1 + \frac{\sin 11^\circ}{\cos 11^\circ}}{1 - \frac{\sin 11^\circ}{\cos 11^\circ}} = \frac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ}$$

$$= R.H.S.$$

Q. Prove that :—

$$\tan 70^\circ = \tan 20^\circ + 2 \tan 50^\circ.$$

Ans:

$$L.H.S. = \tan 70^\circ$$

$$\Rightarrow \tan 70^\circ = \tan(20^\circ + 50^\circ)$$

$$\Rightarrow \tan 70^\circ = \frac{\tan 20^\circ + \tan 50^\circ}{1 - \tan 20^\circ \cdot \tan 50^\circ}$$

$\Rightarrow \tan 70^\circ - \tan 20^\circ \cdot \tan 70^\circ \cdot \tan 50^\circ = \tan 20^\circ + \tan 50^\circ$

$\Rightarrow \tan 70^\circ - \tan(90 - 20^\circ) \cdot \tan 20^\circ \cdot \tan 50^\circ = \tan 20^\circ + \tan 50^\circ$

$\Rightarrow \tan 70^\circ - \cot 20^\circ \cdot \tan 20^\circ \cdot \tan 50^\circ = \tan 20^\circ + \tan 50^\circ$

$\Rightarrow \tan 70^\circ - \tan 50^\circ = \tan 20^\circ + \tan 50^\circ$

$\Rightarrow \tan 70^\circ = \tan 20^\circ + 2 \tan 50^\circ$. proved//

Q. Show that :-

$$\cot(2x) \cdot \cot(x) - \cot(3x) \cdot \cot(2x) - \cot(3x) \cdot \cot(x) = 1.$$

Ans:- we have,

$$\cot(3x) = \cot(2x + x)$$

$$\Rightarrow \cot(3x) = \frac{\cot(2x) \cdot \cot(x) - 1}{\cot(2x) + \cot(x)}$$

$$\Rightarrow \cot(3x) \cdot \cot(2x) + \cot(x) \cdot \cot(3x) = \cot(2x) \cdot \cot(x) - 1$$

$$\Rightarrow \cot(3x) \cdot \cot(2x) + \cot(x) \cdot \cot(3x) - \cot(2x) \cdot \cot(x) = -1$$

$\Rightarrow \cot(2x)\cdot\cot(x) - \cot(3x)\cdot\cot(2x) - \cot(3x)\cdot\cot(x)$
$= 1.$

Q. Find the value of :—

(i) $\cos(840^\circ) = \cos(2\times 360^\circ + 120^\circ)$
$= \cos(120^\circ)$
$= \cos(180^\circ - 60^\circ)$
$= -\cos(60^\circ) = -\frac{1}{2}$

(ii) $\sin(870^\circ) = \sin(2\times 360^\circ + 150^\circ)$
$= \sin(150^\circ)$
$= \sin(180^\circ - 30^\circ)$
$= \sin 30^\circ$
$= \frac{1}{2}.$

(iii) $\tan(-120^\circ) = -\tan(120^\circ)$
$= -\tan(180^\circ - 60^\circ)$
$= -\tan(-\tan(60^\circ))$
$= \tan 60^\circ$
$= \sqrt{3}$

(iv) $\sec(-420^\circ)$

$= \frac{1}{\cos(-420^\circ)} = \frac{1}{-\cos(420^\circ)}$

$= \frac{-1}{\cos(360^\circ + 60^\circ)} = \frac{-1}{\cos 60^\circ} = \frac{-1}{1/2} = -2.$

(v.) $\operatorname{cosec}(-690^\circ)$

Ans: $= \dfrac{1}{\sin(-690^\circ)} = \dfrac{1}{-\sin(690^\circ)}$

$= \dfrac{1}{-\sin(2\times 360^\circ - 30^\circ)}$

$= \dfrac{1}{-(-\sin 30^\circ)} = \dfrac{1}{1/2} = 2$

(vi.) $\tan(225^\circ)$

Ans: $= \tan(180^\circ + 45^\circ)$
$= \tan 45^\circ = 1$

(vii.) $\cot(-315^\circ)$

Ans: $\Rightarrow \dfrac{1}{\tan(-315^\circ)} = \dfrac{1}{-\tan(315^\circ)}$

$= \dfrac{1}{-\tan(360^\circ - 45^\circ)} = \dfrac{1}{+\tan 45^\circ} = +1.$

(viii.) $\sin(-1230^\circ)$

Ans: $= -\sin(1230^\circ)$
$= -\sin(3\times 360^\circ + 150^\circ)$
$= -\sin 150^\circ$
$= -\sin(180^\circ - 30^\circ)$
$= -\sin 30^\circ = -\dfrac{1}{2}$

Q. Prove that :-

(i) $\sin(80^\circ)\cdot\cos(20^\circ) - \cos(80^\circ)\cdot\sin(20^\circ) = \frac{\sqrt{3}}{2}$

Ans:- $\left\{\because \sin A\cdot\cos B - \cos A\cdot\sin B = \sin(A-B)\right\}$

L.H.S.

$= \sin(80^\circ - 20^\circ)$

$= \sin(60^\circ) = \frac{\sqrt{3}}{2}$.

(ii) $\cos(45^\circ)\cdot\cos(15^\circ) - \sin(45^\circ)\cdot\sin(15^\circ) = \frac{1}{2}$

Ans:- $\left\{\because \cos A\cdot\cos B - \sin A\cdot\sin B = \cos(A+B)\right\}$

L.H.S.

$= \cos(45^\circ + 15^\circ) = \cos(60^\circ) = \frac{1}{2}$.

(iii) $\cos(75^\circ)\cdot\cos(15^\circ) + \sin(75^\circ)\cdot\sin(15^\circ) = \frac{1}{2}$

Ans:- $\left\{\because \cos A\cdot\cos B + \sin A\cdot\sin B = \cos(A-B)\right\}$

$= \cos(75^\circ - 15^\circ)$

$= \cos(60^\circ) = \frac{1}{2}$.

(iv) $\sin(40^\circ)\cdot\cos(20^\circ) + \cos(40^\circ)\cdot\sin(20^\circ) = \frac{\sqrt{3}}{2}$

Ans:- $\left\{\because \sin A\cdot\cos B + \cos A\cdot\sin B = \sin(A+B)\right\}$

$= \sin(40^\circ + 20^\circ)$

$= \sin(60^\circ) = \frac{\sqrt{3}}{2}$.

(v.) $\cos 130^\circ \cdot \cos 40^\circ + \sin 130^\circ \cdot \sin 40^\circ = 0$

Ans: L.H.S.

$= \cos(130^\circ - 40^\circ)$

$\left\{ \because \cos A \cdot \cos B + \sin A \cdot \sin B = \cos(A-B) \right\}$

$= \cos(90^\circ) = 0$

Q. Prove that :-

$\sin(50^\circ + \theta) \cdot \cos(20^\circ + \theta) - \cos(50^\circ + \theta) \cdot \sin(20^\circ + \theta) = \frac{1}{2}$.

Ans: $\left\{ \because \sin A \cdot \cos B - \cos A \cdot \sin B = \sin(A-B) \right\}$

$= \sin(50^\circ + \cancel{\theta} - 20^\circ - \cancel{\theta})$

$= \sin(30^\circ)$

$= \frac{1}{2}$.

Q Prove that :-

(i) $\cos(n+2)x \cdot \cos(n+1)x + \sin(n+2)x \cdot \sin(n+1)x = \cos(x)$

Ans: $= \cancel{\sin}\{(n+2)x\} \cdot \cos(n+1)x$

$= \left\{ \because \cos A \cdot \cos B + \sin A \cdot \sin B = \cos(A-B) \right\}$

$= \cos\{(n+2)\cdot x - (n+1)\cdot x\}$

$= \cos\{\cancel{nx} + 2x - \cancel{nx} - x\}$

$= \cos x = R.H.S.$

(ii) $\cos\left(\frac{\pi}{4} - x\right)\cdot\cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - x\right)\cdot\sin\left(\frac{\pi}{4} - y\right)$
$= \sin(x+y)$

Ans: $\{\because \cos A\cdot\cos B - \sin A\cdot\sin B = \cos(A+B)\}$

$= \cos\left(\frac{\pi}{4} - x + \frac{\pi}{4} - y\right)$

$= \cos\left(\frac{\pi}{2} - (x+y)\right) = \sin(x+y).$

Q. Prove that :—

$$\frac{\tan\left(\frac{\pi}{4} + x\right)}{\tan\left(\frac{\pi}{4} - x\right)} = \left(\frac{1+\tan(x)}{1-\tan(x)}\right)^2$$

Ans:

$$L.H.S. = \frac{\dfrac{\tan\left(\frac{\pi}{4}\right) + \tan(x)}{1 - \tan(\pi/4)\cdot\tan(x)}}{\dfrac{\tan\left(\frac{\pi}{4}\right) - \tan(x)}{1 + \tan(\pi/4)\cdot\tan(x)}}$$

$$= \frac{\frac{1+\tan(x)}{1-1\cdot\tan(x)}}{\frac{1-\tan(x)}{1+1\cdot\tan(x)}} = \left(\frac{1+\tan(x)}{1-\tan(x)}\right)^2$$

$\therefore$ L.H.S. = R.H.S.

Q. Prove that :-

$$\sin(75^\circ) = \left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)$$

Ans: L.H.S.

$$= \sin(75^\circ)$$
$$= \sin(90^\circ - 15^\circ)$$
$$= \sin(90^\circ)\cdot\cos(15^\circ) - \cos(90^\circ)\cdot\sin(15^\circ)$$
$$= 1\cdot\cos(15^\circ) - 0\cdot\sin(15^\circ)$$
$$= \cos(15^\circ)$$

$$\cos(15^\circ) = \cos(45^\circ - 30^\circ)$$
$$= \cos 45^\circ \cdot \cos 30^\circ + \sin 45^\circ \cdot \sin 30^\circ$$
$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$
$$= \frac{\sqrt{3}+1}{2\cdot\sqrt{2}} \cdot 1$$
$$= \frac{\sqrt{3}+1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$
$$= \frac{\sqrt{6}+\sqrt{2}}{4}$$

Q. $\frac{\cos(135^\circ) - \cos(120^\circ)}{\cos(135^\circ) + \cos(120^\circ)} = 3 - 2\sqrt{2}$

Ans: L.H.S. =

$$= \frac{\cos(180^\circ - 45^\circ) - \cos(180^\circ - 60^\circ)}{\cos(180^\circ - 45^\circ) + \cos(180^\circ - 60^\circ)}$$

$$= \frac{-\cos(45^\circ) - (-\cos(60^\circ))}{-\cos(45^\circ) + (-\cos(60^\circ))}$$

$$= \frac{\frac{1}{2} - \frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}} + \frac{1}{2}} = \frac{\frac{1-\sqrt{2}}{2}}{\frac{\sqrt{2}+1}{2}}$$

$$= \frac{1-\sqrt{2}}{\sqrt{2}+1} \times \frac{(-\sqrt{2}+1)}{(-\sqrt{2}+1)}$$

$$= \frac{\sqrt{2}+1+2-\sqrt{2}}{-2+\sqrt{2}-\sqrt{2}+1} = \frac{-2\sqrt{2}+3}{-1}$$

$$= 3 - 2\sqrt{2}$$

(P.T.O.)

White Page

Date

Page

(iii) $\tan(15^\circ) + \cot(15^\circ) = 4.$

Ans: L.H.S.

$= \tan(15^\circ) + \cot(15^\circ)$

$\because$ we know,

$$\sin(15^\circ) = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$\cos(15^\circ) = \frac{\sqrt{3}+1}{2\cdot\sqrt{2}}$$

$$= \frac{\frac{\sqrt{3}-1}{\cancel{2\cdot\sqrt{2}}}}{\frac{\sqrt{3}+1}{\cancel{2\sqrt{2}}}} + \frac{\frac{\sqrt{3}+1}{\cancel{2\cdot\sqrt{2}}}}{\frac{\sqrt{3}-1}{\cancel{2\cdot\sqrt{2}}}}$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} + \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$= \frac{(\sqrt{3}-1)^2 + (\sqrt{3}+1)^2}{(\sqrt{3}+1)\cdot(\sqrt{3}-1)}$$

$$= \frac{3+1-\cancel{2\sqrt{3}}+3+1+\cancel{2\sqrt{3}}}{3-1}$$

$$= \frac{8}{2} = 4$$ proved

Q. Prove that :—

$$\cos(15^\circ) - \sin(15^\circ) = \frac{1}{\sqrt{2}}$$

Ans: L.H.S.

$$= \cos(15^\circ) - \sin(15^\circ)$$

$\because$ we know that,

$$\sin(15^\circ) = \frac{\sqrt{3}-1}{2\sqrt{2}} \; ; \; \cos(15^\circ) = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3}+1}{2\cdot\sqrt{2}} - \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$= \frac{\cancel{\sqrt{3}}+1-\cancel{\sqrt{3}}+1}{2\cdot\sqrt{2}} = \frac{\cancel{2}}{\cancel{2}\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$= R.H.S$

Q. Prove that :—

$$\cot(105^\circ) - \tan(105^\circ) = 2\cdot\sqrt{3}$$

Ans: L.H.S.

$$= \cot(180^\circ - 75^\circ) - \tan(180^\circ - 75^\circ)$$

$$= -\cot(75^\circ) - (-\tan(75^\circ))$$

$$= \tan(75^\circ) - \cot(75^\circ)$$

$\because$ we know,
$$\tan 75^\circ = \frac{-\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{-\sqrt{3}-1}{\sqrt{3}-1} - \frac{\sqrt{3}-1}{-\sqrt{3}-1}$$

$$= \frac{-\cancel{\sqrt{3}}-1+\cancel{\sqrt{3}}-1}{(\sqrt{3}-1)(1+\sqrt{3})}$$

$$= \frac{-2}{\cancel{3}-\cancel{1}} = \cancel{-2}$$

$$= \frac{-2}{\sqrt{3}+3-1-\sqrt{3}}$$

$$= \frac{\sqrt{3}-1}{-\sqrt{3}-1} - \frac{-\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{(\sqrt{3}-1)^2 - (-\sqrt{3}-1)^2}{(-\sqrt{3}-1)(\sqrt{3}-1)}$$

$$= \frac{3+1-2\sqrt{3}-(3+1+2\sqrt{3})}{(-3+1-\cancel{\sqrt{3}}+\cancel{\sqrt{3}})}$$

$$= \frac{\cancel{4} - 2\sqrt{3} - \cancel{4} - 2\sqrt{3}}{-2}$$

$$= \frac{-\cancel{4}^{2}\sqrt{3}}{-\cancel{2}} = 2\sqrt{3}.$$

Q. prove that :-

$$\frac{\tan 69^\circ + \tan 66^\circ}{1 - \tan(69^\circ) \cdot \tan(66^\circ)} = -1.$$

Ans :-

$$\left\{ \because \frac{\tan(A) + \tan(B)}{1 - \tan(A) \cdot \tan(B)} = \tan(A+B) \right\}$$

$$= \tan(69^\circ + 66^\circ)$$

$$= \tan(135^\circ) = -1$$

Q. Prove that :-

$$\frac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ} = \tan(54^\circ)$$

Ans :- L.H.S.

$$= \frac{\cancel{\cos 9^\circ}(1 + \tan 9^\circ)}{\cancel{\cos 9^\circ}(1 - \tan 9^\circ)} = \frac{\tan 45^\circ + \tan 9^\circ}{1 - \tan 45^\circ \cdot \tan 9^\circ}$$

$$= \tan(45^\circ + 9^\circ)$$

$$\left\{ \because \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} = \tan(A+B) \right\} \quad = \tan(54^\circ) = \text{R.H.S}$$

White Page

Date

Page

Q. Prove that :-

$$\frac{\cos 8^\circ - \sin 8^\circ}{\cos 8^\circ + \sin 8^\circ} = \tan(37^\circ).$$

Ans:- L.H.S.

$$= \frac{\cancel{\cos 8^\circ}(1 - \tan 8^\circ)}{\cancel{\cos 8^\circ}(1 + \tan 8^\circ)}$$

$$= \frac{\tan 45^\circ - \tan 8^\circ}{1 + \tan 45^\circ \cdot \tan 8^\circ} = \tan(45^\circ - 8^\circ)$$

$$= \tan(37^\circ)$$

$$= R.H.S.$$

Q. Prove that :-

$$\frac{\cos(\pi + \theta) \cdot \cos(-\theta)}{\cos(\pi - \theta) \cdot \cos\left(\frac{\pi}{2} + \theta\right)} = -\cot(\theta)$$

$$L.H.S. = \frac{-\cos(\theta) \cdot \cos(\theta)}{\cancel{-}\cos\theta \cdot \cancel{-}\sin\theta} = -\cot\theta.$$

Q. Prove that :-

$$\frac{\cos\theta}{\sin(90^\circ + \theta)} + \frac{\sin(-\theta)}{\sin(180^\circ + \theta)} - \frac{\tan(90^\circ + \theta)}{\cot(\theta)} = 3$$

Ans: L.H.S.

$$= \frac{\cancel{\cos(\theta)}}{\cancel{\cos(\theta)}} + \frac{\cancel{-\sin(\theta)}}{\cancel{-\sin(\theta)}} + \frac{\cancel{\cot(\theta)}}{\cancel{\cot(\theta)}}$$

$$= 1 + 1 + 1 = 3. = R.H.S.$$

Q. Prove that :—

$$\frac{\sin(180^\circ+\theta)\cdot\cos(90^\circ+\theta)\cdot\tan(270^\circ-\theta)\cdot\cot(360^\circ-\theta)}{\sin(360^\circ-\theta)\cdot\cos(360^\circ+\theta)\cdot\operatorname{cosec}(-\theta)\cdot\sin(270^\circ+\theta)} = 1$$

Ans: L.H.S.

$$= \frac{(\cancel{-}\sin(\theta))\times(\cancel{-}\sin(\theta))\times\cot(\theta)\times(\cancel{-}\,\overset{\cot}{\cancel{\tan}}(\theta))}{(\cancel{-}\sin(\theta))\times\cos(\theta)\times\cancel{-}\operatorname{cosec}(\theta)\times\cancel{-}\cos(\theta)}$$

$$= \frac{\cancel{\sin^2\theta}\times\cancel{\cos^2\theta}}{\cancel{\sin^2\theta}\times\cancel{\cos^2\theta}} = 1 = R.H.S.$$

Q. Prove that :—

(i) $\cos\left(\frac{\pi}{3}+x\right) = \frac{1}{2}\cdot\left(\cos(x) - \sqrt{3}\cdot\sin(x)\right)$

Ans: $\left\{\because \cos(A+B) = \cos(A)\cdot\cos(B) - \sin(A)\cdot\sin(B)\right\}$

$$= \frac{1}{2}\cdot\cos(x) - \frac{\sqrt{3}}{2}\cdot\sin(x)$$

$$= \frac{1}{2}\left(\cos(x) - \sqrt{3}\cdot\sin(x)\right)$$

$$= R.H.S.$$

(ii) $\sin\left(\frac{\pi}{4}+x\right)+\sin\left(\frac{\pi}{4}-x\right)=\sqrt{2}\cdot\cos(x)$

Ans: $\left\{\begin{array}{l} \because \sin(A+B)=\sin A\cdot\cos B+\cos A\cdot\sin B \\ \sin(A-B)=\sin A\cdot\cos B-\cos A\cdot\sin B \end{array}\right\}$

L.H.S.

$=\sin\left(\frac{\pi}{4}\right)\cdot\cos(x)+\cancel{\cos\left(\frac{\pi}{4}\right)\cdot\sin(x)}$
$+\sin\left(\frac{\pi}{4}\right)\cdot\cos(x)-\cancel{\cos\left(\frac{\pi}{4}\right)\cdot\sin(x)}$

$=2\cdot\sin\left(\frac{\pi}{4}\right)\cos(x)$

$=2\times\frac{1}{\sqrt{2}}\times\cos(x)$

$=\frac{\cancel{2}\times\sqrt{2}}{\cancel{\sqrt{2}}\,\cancel{\sqrt{2}}}\times\cos(x)=\sqrt{2}\cdot\cos(x)$
$=$ R.H.S.

(iii) $\frac{1}{\sqrt{2}}\cdot\cos\left(\frac{\pi}{4}+x\right)=\frac{1}{2}\left(\cos(x)-\sin(x)\right)$

Ans: L.H.S. $\left\{\because \cos(A+B)=\cos A\cdot\cos B-\sin A\cdot\sin B\right\}$

$=\frac{1}{\sqrt{2}}\left(\cos\frac{\pi}{4}\cdot\cos(x)-\sin\left(\frac{\pi}{4}\right)\cdot\sin(x)\right)$

$=\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\cdot\cos(x)-\frac{1}{\sqrt{2}}\cdot\sin(x)\right)$

$=\frac{1}{\sqrt{2}\times\sqrt{2}}\left[\cos(x)-\sin(x)\right]$

White Page

Date ______

Page ______

$= \frac{1}{2}\left(\cos(x) - \sin(x)\right)$

(iv) $\cos(x) + \cos\left(\frac{2\pi}{3} + x\right) + \cos\left(\frac{2\pi}{3} - x\right) = 0$

Ans: $\left(\because \cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B\right)$

L.H.S:

$= \cos(x) + \cos\left(\frac{2\pi}{3}\right)\cdot\cos(x) - \cancel{\sin\left(\frac{2\pi}{3}\right)\cdot\sin(x)}$

$+ \cos\left(\frac{2\pi}{3}\right)\cdot\cos(x) + \cancel{\sin\left(\frac{2\pi}{3}\right)\cdot\sin(x)}$

$= \cos(x) + 2\cdot\cos\left(\frac{2\pi}{3}\right)\cdot\cos(x)$

$= \cos(x) + 2\cdot\cos\left(\pi - \frac{\pi}{3}\right)\cdot\cos(x)$

$= \cos(x) + \cancel{2}\times\left(-\frac{1}{\cancel{2}}\right)\cdot\cos(x) = 0$

Q. Prove that :-

(i) $2\cdot\sin\left(\frac{5\pi}{12}\right)\cdot\sin\left(\frac{\pi}{12}\right) = \frac{1}{2}$

L.H.S =

$\left\{\because 2\cdot\sin(A)\cdot\sin(B) = \cos(A-B) - \cos(A+B)\right\}$

$= -\left(\cos\left(\frac{5\pi}{12} + \frac{\pi}{12}\right) - \cos\left(\frac{5\pi}{12} - \frac{\pi}{12}\right)\right)$

$= -\left(\cos\left(\frac{6\pi}{12}\right) - \cos\left(\frac{4\pi}{12}\right)\right)$

$= -\left(\cos\left(\frac{\pi}{2}\right) - \cos\left(\frac{\pi}{3}\right)\right)$

$$= -\left(0 - \frac{1}{2}\right) = \frac{1}{2} = R.H.S.$$

(ii) $2 \cdot \cos\left(\frac{5\pi}{12}\right) \cdot \cos\left(\frac{\pi}{12}\right) = \frac{1}{2}$

Ans: L.H.S.

$$= \left\{ \because 2 \cdot \cos A \cdot \cos B = \cos(A+B) + \cos(A-B) \right\}$$

$$= \cos\left(\frac{5\pi}{12} + \frac{\pi}{12}\right) + \cos\left(\frac{5\pi}{12} - \frac{\pi}{12}\right)$$

$$= \cos\left(\frac{\cancel{6}\pi}{\cancel{12}2}\right) + \cos\left(\frac{\cancel{4}\pi}{\cancel{12}3}\right)$$

$$= \cos\left(\frac{\pi}{2}\right) + \cos\left(\frac{\pi}{3}\right)$$

$$= 0 + \frac{1}{2} = \frac{1}{2} = R.H.S.$$

Q. If $\sin(\theta) = \frac{15}{17}$ and $\cos(\theta) = \frac{12}{13}$, where θ and ϕ both lie in the first quadrant, find the values of :-

(i) $\sin(\theta + \phi)$
(ii) $\cos(\theta - \phi)$
(iii) $\tan(\theta + \phi)$

(P.T.O.)

Ans:

$\because \sin(\theta) = \frac{15}{17}$ & $\cos(\phi) = \frac{12}{13}$

$\because$ θ lies in the first quadrant, we have $\sin(\theta) > 0$, $\cos(\theta) > 0$ and $\tan(\theta) > 0$.

again,

$\because$ ϕ lies in the first quadrant, we have $\sin(\phi) > 0$, $\cos(\phi) > 0$ and $\tan(\phi) > 0$.

Now,

$$\cos^2\theta = 1 - \sin^2(\theta)$$

$$= \left(1 - \frac{225}{289}\right) = \frac{64}{289}$$

$$\cos\theta = +\sqrt{\frac{64}{289}} = \frac{8}{17}$$

And,

$$\sin^2\phi = (1 - \cos^2\phi)$$

$$= \left(1 - \frac{144}{169}\right) = \frac{25}{169}$$

$$\Rightarrow \sin\phi = +\sqrt{\frac{25}{169}} = \frac{5}{13}$$

$\therefore$

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{15}{\cancel{17}} \times \frac{\cancel{17}}{8} = \frac{15}{8} \quad (1)$$

$$\tan(\phi) = \frac{\sin(\phi)}{\cos(\phi)} = \frac{5}{\cancel{13}} \times \frac{\cancel{13}}{12} = \frac{5}{12}$$

(P.T.O.)

Now,

(i.) $\sin(\theta + \phi)$

$= \sin\theta \cdot \cos\phi + \cos\theta \cdot \sin\phi$

$= \left(\frac{15}{17} \times \frac{12}{13}\right) + \left(\frac{8}{17} \times \frac{5}{13}\right)$

$= \left(\frac{180}{221} + \frac{40}{221}\right)$

$= \frac{220}{221}$

(ii.) $\cos(\theta - \phi)$

$= \cos\theta \cdot \cos\phi + \sin\theta \cdot \sin\phi$

$= \left(\frac{8}{17} \times \frac{12}{13}\right) + \left(\frac{15}{17} \times \frac{5}{13}\right)$

$= \left(\frac{96}{221} + \frac{75}{221}\right) = \frac{171}{221}$

(iii.) $\tan(\theta + \phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta \cdot \tan\phi}$

$\Rightarrow \frac{\left(\frac{15}{8} + \frac{5}{12}\right)}{\left\{1 - \left(\frac{15}{8} \times \frac{5}{12}\right)\right\}} = \frac{\frac{55}{24}}{1 - \frac{25}{32}}$

$\Rightarrow \left(\frac{55}{24} \times \frac{32}{7}\right) = \frac{220}{21}$

Q. If $\cot(\alpha) = \frac{1}{2}$ and $\sec(\beta) = -\frac{5}{3}$, where $\alpha \in \left]\pi, \frac{3\pi}{2}\right[$ and $\beta \in \left]\frac{\pi}{2}, \pi\right[$, find the value of $\tan(\alpha+\beta)$.

Ans: Here, α lies in Quadrant III &, therefore, $\tan(\alpha)$ is positive.

Now,

$$\cot(\alpha) = \frac{1}{2} \Rightarrow \tan(\alpha) = 2.$$

again,

β lies in Quadrant II and, therefore, $\sin(\beta)$ is positive and $\cos(\beta)$ is negative

Now,

$$\sec(\beta) = -\frac{5}{3} \Rightarrow \cos(\beta) = -\frac{3}{5}.$$

and,

$$\sin(\beta) = +\sqrt{1-\cos^2(\beta)}$$

$$= +\sqrt{1-\frac{9}{25}} = +\sqrt{\frac{16}{25}} = \frac{4}{5}$$

$$\therefore \tan(\beta) = \frac{\sin(\beta)}{\cos(\beta)} = \frac{4}{5} \times \left(-\frac{5}{3}\right) = -\frac{4}{3}$$

$$\therefore \tan(\alpha+\beta) = \frac{\tan(\alpha)+\tan(\beta)}{1-\tan(\alpha)\cdot\tan(\beta)}$$

$$\Rightarrow \frac{2-\frac{4}{3}}{1-2\times\left(-\frac{4}{3}\right)} = \frac{\left(\frac{2}{3}\right)}{\left(\frac{11}{3}\right)}$$

$$\Rightarrow \frac{2}{\cancel{3}} \times \frac{\cancel{3}}{11} = \frac{2}{11} = \text{R}$$

Q. If x and y are acute angles such that $\sin(x) = \frac{1}{\sqrt{5}}$ and $\sin(y) = \frac{1}{\sqrt{10}}$, prove that $(x+y) = \frac{\pi}{4}$.

Ans: given,

$$\sin(x) = \frac{1}{\sqrt{5}} \ \& \ \sin(y) = \frac{1}{\sqrt{10}}$$

$$\cos(x) = \sqrt{1-\sin^2(x)}$$

$$= \sqrt{1-\left(\frac{1}{\sqrt{5}}\right)^2}$$

$$= \sqrt{\frac{5-1}{5}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$$

$$\cos \ \cancel{\sin}(y) = \sqrt{1-\sin^2(y)}$$

$$= \sqrt{1-\left(\frac{1}{\sqrt{10}}\right)^2}$$

$$= \sqrt{\frac{10-1}{10}} = \frac{3}{\sqrt{10}}$$

$\because \sin(x+y) = \sin(x)\cdot\cos(y) + \cos(x)\cdot\sin(y)$

$$= \frac{1}{\sqrt{5}} \times \frac{3}{\sqrt{10}} + \frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{10}}$$

$$= \frac{3}{\sqrt{50}} + \frac{2}{\sqrt{50}} = \frac{5}{\sqrt{50}} = \frac{\cancel{5}}{\cancel{5}\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}$$

$\Rightarrow \sin(x+y) = \frac{1}{\sqrt{2}}$

$\Rightarrow x+y = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$\Rightarrow x+y = \frac{\pi}{4}$ proved

Q. If x and y are acute angles such that $\cos(x) = \frac{13}{14}$ and $\cos(y) = \frac{1}{7}$ prove that $(x-y) = -\frac{\pi}{3}$.

Ans:- given,

$$\cos(x) = \frac{13}{14} \text{ and } \cos(y) = \frac{1}{7}.$$

$\sin(x) = \sqrt{1-\cos^2(x)}$

(P.T.O.)

$$= \sqrt{1-\left(\frac{13}{14}\right)^2} \Rightarrow \sqrt{\frac{196-169}{196}}$$

$$= \sqrt{\frac{27}{196}} = \frac{3\sqrt{3}}{14}.$$

$$\sin(y) = \sqrt{1-\cos^2(y)}$$

$$= \sqrt{1-\left(\frac{1}{7}\right)^2} = \sqrt{\frac{49-1}{49}}$$

$$= \sqrt{\frac{48}{49}} = \frac{4\sqrt{3}}{7}$$

<u>hence,</u>

$$\cos(x-y) = \cos(x)\cdot\cos(y) + \sin(x)\cdot\sin(y)$$

$$= \frac{13}{14}\times\frac{1}{7} + \frac{3\sqrt{3}}{14}\times\frac{4\sqrt{3}}{7}$$

$$= \frac{13+36}{98} = \frac{49}{98}$$

$$\cos(x-y) = \frac{1}{2}$$

$$(x-y) = \frac{\pi}{3}$$

White Page
Date
Page

Q. Prove that :—

$$\cos\left(\frac{\pi}{4}+x\right)+\cos\left(\frac{\pi}{4}-x\right)=\sqrt{2}\cdot\cos(x)$$

Ans:

L.H.S.

$$=\left\{\because \cos(A+B)+\cos(A-B)=2\cdot\cos A\cdot\cos B\right\}$$

$$=2\cdot\cos\left(\frac{\pi}{4}\right)\cdot\cos(x)$$

$$=\left(2\times\frac{1}{\sqrt{2}}\cdot\cos(x)\right)=\frac{\cancel{2}}{\sqrt{2}}\times\frac{\sqrt{2}}{\cancel{\sqrt{2}}}\times\cos(x)$$

$$=\sqrt{2}\cdot\cos(x)=\text{R.H.S.}$$

Q. Prove that :—

$$\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)=-\sqrt{2}\cdot\sin(x)$$

Ans:

$$\left\{\because \cos(A+B)-\cos(A-B)=-2\cdot\sin A\cdot\sin B\right\}$$

$$=-2\cdot\sin\left(\frac{3\pi}{4}\right)\cdot\sin(x)$$

$$=-2\cdot\sin\left(\pi-\frac{\pi}{4}\right)\cdot\sin(x)$$

$$=-2\cdot\sin\frac{\pi}{4}\cdot\sin(x)$$

$$=\left(2\cdot\times\frac{1}{\sqrt{2}}\right)\cdot\sin(x)$$

$$=-\sqrt{2}\cdot\sin(x)=\text{R.H.S.}$$

Q. Express each of the following as an algebraic sum of sines or cosines.

(i) $2\cdot\sin(3x)\cdot\cos(2x)$

Ans: $\left\{\because 2\cdot\sin(A)\cdot\cos(B)=\sin(A+B)+\sin(A-B)\right\}$

$= \sin(3x+2x)+\sin(3x-2x)$

$= \sin(5x)+\sin(x)$

(ii) $2\cdot\cos(4x)\cdot\overset{\cos}{\cancel{\sin}}(2x)$

Ans: $\left\{\because 2\cdot\cos(A)\cdot\cos(B)=\cos(A+B)+\cos(A-B)\right\}$

$= \cos(4x+2x)+\cos(4x-2x)$

$= \cos(6x)+\cos(2x)$

(iii) $2\cdot\cos(6x)\cdot\cos(4x)$

Ans: $\left\{\because 2\cdot\cos(A)\cdot\cos(B)=\cos(A+B)+\cos(A-B)\right\}$

$= \cos(6x+4x)+\cos(6x-4x)$

$= \cos(10x)+\cos(2x)$

(iv) $2\cdot\sin(3x)\cdot\sin(5x)$

Ans: $\left\{\because 2\sin A\cdot\sin B=\cos(A-B)-\cos(A+B)\right\}$

$= 2\cdot\sin(5x)\cdot\sin(3x)$

(iii) $\sin(7x) + \sin(3x)$

Ans: $\because \sin C + \sin D = 2 \cdot \sin\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right)$

$\Rightarrow 2 \cdot \sin\left(\frac{7x+3x}{2}\right) \cdot \cos\left(\frac{7x-3x}{2}\right)$

$\Rightarrow 2 \cdot \sin(5x) \cdot \cos(2x)$

(iv) $\sin(5x) - \sin(3x)$

Ans: $\because \sin C - \sin D = 2 \cdot \cos\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right)$

$= 2 \cdot \cos\left(\frac{5x+3x}{2}\right) \cdot \sin\left(\frac{5x-3x}{2}\right)$

$= 2 \cdot \cos(4x) \cdot \sin(x)$.

Q. Prove that :—

$$\frac{\cos(6x) + \cos(4x)}{\sin(6x) - \sin(4x)} = \cot(x)$$

Ans: L.H.S.

$$= \frac{\cos(6x) + \cos(4x)}{\sin(6x) - \sin(4x)}$$

$$= \frac{2 \cdot \cos\left(\frac{6x+4x}{2}\right) \cdot \cos\left(\frac{6x-4x}{2}\right)}{2 \cdot \cos\left(\frac{6x+4x}{2}\right) \cdot \sin\left(\frac{6x-4x}{2}\right)}$$

$$\left\{\begin{array}{l} \because \cos C + \cos D = 2\cdot\cos\left(\frac{C+D}{2}\right)\cdot\cos\left(\frac{C-D}{2}\right) \\ \sin C - \sin D = 2\cdot\cos\left(\frac{C+D}{2}\right)\cdot\sin\left(\frac{C-D}{2}\right) \end{array}\right\}$$

$$= \frac{\cancel{2}\cdot\cancel{\cos(5x)}\cdot\cos(x)}{\cancel{2}\cdot\cancel{\cos(5x)}\cdot\sin(x)} = \cot(x).$$

$$= R.H.S.$$

Q. Prove that :—

$$\frac{\sin(3x) - \sin(x)}{\cos(x) - \cos(3x)} = \cot(2x)$$

Ans: L.H.S.

$$= \frac{\sin(3x) - \sin(x)}{\cos(x) - \cos(3x)}$$

$$= \left\{\begin{array}{l} \because \sin C - \sin D = 2\cdot\cos\left(\frac{C+D}{2}\right)\cdot\sin\left(\frac{C-D}{2}\right) \\ \cos C - \cos D = -2\cdot\sin\left(\frac{C+D}{2}\right)\cdot\sin\left(\frac{C-D}{2}\right) \end{array}\right\}$$

$$= \frac{2\cdot\cos\left(\frac{3x+x}{2}\right)\cdot\sin\left(\frac{3x-x}{2}\right)}{-2\cdot\sin\left(\frac{x+3x}{2}\right)\cdot\sin\left(\frac{x-3x}{2}\right)}$$

$$\Rightarrow \frac{\cos(2x)\cdot\sin(x)}{-\sin(2x)\cdot\sin(-x)} = \frac{\cos(2x)\cdot\cancel{\sin(x)}}{\sin(2x)\cdot\cancel{\sin(x)}}$$

$$= \cot(2x) = R.H.S.$$

Q. Prove that :—

$$\frac{\sin(3x) - \sin(x)}{\cos(2x)} = 2 \cdot \sin(x)$$

Ans :

L.H.S.

$$= \left\{ \because \sin(C) - \sin(D) = 2 \cdot \cos\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right) \right\}$$

$$= \frac{2 \cdot \cos\left(\frac{3x+x}{2}\right) \cdot \sin\left(\frac{3x-x}{2}\right)}{\cos(2x)}$$

$$= \frac{2 \cdot \cancel{\cos(2x)} \cdot \sin(x)}{\cancel{\cos(2x)}} = 2 \cdot \sin(x)$$

$$= R.H.S.$$

Q. Prove that :—

$$\frac{\sin(5x) - 2 \cdot \sin(3x) + \sin(x)}{\cos(5x) - \cos(x)} = \operatorname{cosec}(2x) - \cot(2x)$$

Ans : L.H.S.

$$\left\{ \begin{array}{l} \because \sin C + \sin D = 2 \cdot \sin\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right) \\ \cos C - \cos D = 2 \cdot \sin\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right) \end{array} \right\}$$

$$= \frac{\left(\sin(5x) + \sin(x)\right) - 2 \cdot \sin(3x)}{\cos(5x) - \cos(x)}$$

$$= \frac{2\cdot \sin\left(\frac{5x+x}{2}\right)\cdot \cos\left(\frac{5x-x}{2}\right) - 2\sin(3x)}{-2\cdot \sin\left(\frac{5x+x}{2}\right)\cdot \sin\left(\frac{5x-x}{2}\right)}$$

$$= \frac{2\cdot \sin(3x)\cdot \cos(2x) - 2\cdot \sin(3x)}{-2\cdot \sin(3x)\cdot \sin(2x)}$$

$$= \frac{\cancel{2}\,\cancel{\sin(3x)}\,[\cos(2x) - 1]}{-\cancel{2}\cdot \cancel{\sin(3x)}\cdot \sin(2x)}$$

$$= \frac{1-\cos(2x)}{\sin(2x)} = \frac{1}{\sin(2x)} - \frac{\cos(2x)}{\sin(2x)}$$

$$= \operatorname{cosec}(2x) - \cot(2x)$$
$= R.H.S.$

Q. If $\frac{\cos(A+B)}{\cos(A-B)} = \frac{\sin(C+D)}{\sin(C-D)}$, prove that

$\tan(A)\cdot\tan(B)\cdot\tan(C) + \tan D = 0.$

Ans: ~~L.H.S.~~

We have,

$$\frac{\cos(A+B)}{\cos(A-B)} = \frac{\sin(C+D)}{\sin(C-D)}$$

$$\Rightarrow \frac{\cos(A+B)+\cos(A-B)}{\cos(A+B)-\cos(A-B)} = \frac{\sin(C+D)+\sin(C-D)}{\sin(C+D)-\sin(C-D)}$$

$$\Rightarrow \frac{2\cdot\cos(A)\cdot\cos(B)}{-2\cdot\sin(A)\cdot\sin(B)} = \frac{2\cdot\sin(C)\cdot\cos(D)}{2\cdot\cos(C)\cdot\sin(D)}$$

$$\Rightarrow -\cot(A)\cdot\cot(B) = \tan(C)\cdot\cot D$$

$$\Rightarrow \tan A\cdot\tan B\cdot\tan C = -\tan D$$

$$\Rightarrow \tan A\cdot\tan B\cdot\tan C + \tan D = 0$$

proved!

Q. Prove that :—

$$\frac{\cos(4x)+\cos(3x)+\cos(2x)}{\sin(4x)+\sin(3x)+\sin(2x)} = \cot(3x)$$

Ans: We have,

$$L.H.S. = \frac{(\cos(4x)+\cos(2x))+\cos(3x)}{(\sin(4x)+\sin(2x))+\sin(3x)}$$

$$= \frac{2\cdot\cos\left(\frac{4x+2x}{2}\right)\cdot\cos\left(\frac{4x-2x}{2}\right)+\cos(3x)}{2\cdot\sin\left(\frac{4x+2x}{2}\right)\cdot\cos\left(\frac{4x-2x}{2}\right)+\sin(3x)}$$

$$\left\{\begin{array}{l} \because \cos C+\cos D = 2\cdot\cos\left(\frac{C+D}{2}\right)\cdot\cos\left(\frac{C-D}{2}\right) \\ \sin C+\sin D = 2\cdot\sin\left(\frac{C+D}{2}\right)\cdot\cos\left(\frac{C-D}{2}\right) \end{array}\right\}$$

$$= \frac{2\cdot \cos(3x)\cdot \cos(x) + \cos(3x)}{2\cdot \sin(3x)\cdot \cos(x) + \sin(3x)}$$

$$= \frac{\cos(3x)\{\cancel{2\cdot \cos(x) + 1}\}}{\sin(3x)\{\cancel{2\cdot \cos(x) + 1}\}}$$

$$= \frac{\cos(3x)}{\sin(3x)} = \cot(3x) = R.H.S.$$

Q. Prove that :—

$\sin(x) + \sin(3x) + \sin(5x) + \sin(7x) =$
$4\sin(4x)\cdot \cos(2x)\cdot \cos(x)$.

Ans: L.H.S.

$= (\sin(7x) + \sin(x)) + (\sin(5x) + \sin(3x))$

$= 2\cdot \sin\left(\frac{7x+x}{2}\right)\cdot \cos\left(\frac{7x-x}{2}\right) +$
$2\cdot \sin\left(\frac{5x+3x}{2}\right)\cdot \cos\left(\frac{5x-3x}{2}\right)$

$= 2\cdot \sin(4x)\cdot \cos(3x) + 2\cdot \sin(4x)\cdot \cos(x)$

$= 2\cdot \sin(4x) \times \{\cos(3x) + \cos(x)\}$

$= 2\sin(4x) \times 2\cdot \cos\left(\frac{3x+x}{2}\right)\cdot \cos\left(\frac{3x-x}{2}\right)$

$= 2\sin(4x) \times 2\cdot \cos(2x)\cdot \cos(x)$

$= 4\cdot \sin(4x)\cdot \cos(2x)\cdot \cos(x)$

$= R.H.S.$

Q. Prove that:-

$$\cos(2x)\cdot\cos\left(\frac{x}{2}\right)-\cos(3x)\cdot\cos\left(\frac{9x}{2}\right)=-\sin(5x)\cdot\sin\left(\frac{5x}{2}\right)$$

Ans: L.H.S.

$$=\frac{1}{2}\left[2\cdot\cos(2x)\cdot\cos\left(\frac{x}{2}\right)-2\cdot\cos(3x)\cdot\cos\left(\frac{9x}{2}\right)\right]$$

$$=\frac{1}{2}\left[\cos\left(2x+\frac{x}{2}\right)+\cos\left(2x-\frac{x}{2}\right)\right]-\frac{1}{2}\left[\cos\left(\frac{9x}{2}+3x\right)+\cos\left(\frac{9x}{2}-3x\right)\right]$$

$$=\frac{1}{2}\left[\cos\left(\frac{5x}{2}\right)+\cos\left(\frac{3x}{2}\right)\right]-\frac{1}{2}\left[\cos\left(\frac{15x}{2}\right)+\cos\left(\frac{3x}{2}\right)\right]$$

$$=\frac{1}{2}\left[\cos\left(\frac{5x}{2}\right)-\cos\left(\frac{15x}{2}\right)\right]$$

$$=\frac{1}{\cancel{2}}\left[-\cancel{2}\cdot\sin\frac{\left(\frac{5x}{2}+\frac{15x}{2}\right)}{2}\cdot\sin\frac{\left(\frac{15x}{2}-\frac{5x}{2}\right)}{2}\right]$$

$$=-\sin(5x)\cdot\sin\left(\frac{5x}{2}\right)$$

$= R.H.S.$

Q. Prove that :—

$$\frac{\sin(8x)\cdot\cos(x)-\sin(6x)\cdot\cos(3x)}{\cos(2x)\cdot\cos(x)-\sin(4x)\cdot\sin(3x)}=\tan(2x)$$

Ans: L.H.S.

Multiplying n^r & d^r by '2', we get

$$=\frac{2\cdot\sin(8x)\cdot\cos(x)-2\sin(6x)\cdot\cos(3x)}{2\cdot\cos(2x)\cdot\cos(x)-2\sin(4x)\cdot\sin(3x)}$$

$$\left\{\begin{array}{l}\because\ 2\sin A\cdot\cos B=\sin(A+B)+\sin(A-B)\\ 2\cdot\cos A\cdot\cos B=\cos(A+B)+\cos(A-B)\\ 2\cdot\sin A\cdot\sin B=\cos(A-B)-\cos(A+B)\end{array}\right.$$

$$=\frac{\{\sin(8x+x)+\sin(8x-x)\}-\{\sin(6x+3x)+\sin(6x-3x)\}}{\{\cos(2x+x)+\cos(2x-x)\}-\{\cos(4x-3x)-\cos(4x+3x)\}}$$

$$=\frac{\{\sin(9x)+\sin(7x)\}-\{\sin(9x)+\sin(3x)\}}{\{\cos(3x)+\cos(x)\}-\{\cos(x)-\cos(7x)\}}$$

$$=\frac{\cancel{\sin(9x)}+\sin(7x)-\cancel{\sin(9x)}-\sin(3x)}{\cos(3x)+\cancel{\cos(x)}-\cancel{\cos(x)}+\cos(7x)}$$

$$=\frac{\sin(7x)-\sin(3x)}{\cos(3x)+\cos(7x)}$$

$$\left\{ \begin{array}{l} \because \sin C - \sin D = 2 \cdot \cos\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right) \\ \cos C + \cos D = 2 \cdot \cos\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right) \end{array} \right.$$

$$= \frac{\cancel{2} \cdot \cos\left(\frac{7x+3x}{2}\right) \cdot \sin\left(\frac{7x-3x}{2}\right)}{\cancel{2} \cdot \cos\left(\frac{7x+3x}{2}\right) \cdot \cos\left(\frac{7x-3x}{2}\right)}$$

$$= \frac{\cancel{\cos(5x)} \cdot \sin(2x)}{\cancel{\cos(5x)} \cdot \cos(2x)} = \tan(2x) = R.H.S.$$

Q. Prove that :-

$$\frac{\sin(x) - \sin(y)}{\cos(x) + \cos(y)} = \tan\left(\frac{x-y}{2}\right)$$

Ans: L.H.S.

$$= \frac{\sin(x) - \sin(y)}{\cos(x) + \cos(y)}$$

$$= \frac{\cancel{2} \cdot \cancel{\cos\left(\frac{x+y}{2}\right)} \cdot \sin\left(\frac{x-y}{2}\right)}{\cancel{2} \cdot \cancel{\cos\left(\frac{x+y}{2}\right)} \cdot \cos\left(\frac{x-y}{2}\right)}$$

$$= \tan\left(\frac{x-y}{2}\right) = R.H.S.$$

Q. Prove that :—

$$\left(\cos(x)+\cos(y)\right)^2+\left(\sin(x)+\sin(y)\right)^2=4\cdot\cos^2\left(\frac{x-y}{2}\right)$$

Ans: L.H.S.

$$=\left(\cos(x)+\cos(y)\right)^2+\left(\sin(x)+\sin(y)\right)^2$$

$$\left\{\begin{array}{l}\because \cos C+\cos D=2\cdot\cos\left(\frac{C+D}{2}\right)\cdot\cos\left(\frac{C-D}{2}\right)\\ \sin C+\sin D=2\cdot\sin\left(\frac{C+D}{2}\right)\cdot\cos\left(\frac{C-D}{2}\right)\end{array}\right\}$$

$$=\left\{2\cdot\cos\left(\frac{x+y}{2}\right)\cdot\cos\left(\frac{x-y}{2}\right)\right\}^2+$$

$$\left\{2\cdot\sin\left(\frac{x+y}{2}\right)\cdot\cos\left(\frac{x-y}{2}\right)\right\}^2$$

$$=4\cdot\cos^2\left(\frac{x+y}{2}\right)\cdot\cos^2\left(\frac{x-y}{2}\right)+$$

$$4\cdot\sin^2\left(\frac{x+y}{2}\right)\cdot\cos^2\left(\frac{x-y}{2}\right)$$

$$=4\cdot\cos^2\left(\frac{x-y}{2}\right)\left\{\cos^2\left(\frac{x+y}{2}\right)+\sin^2\left(\frac{x+y}{2}\right)\right\}$$

$$=4\cdot\cos^2\left(\frac{x-y}{2}\right)\qquad\left[\because \sin^2\theta+\cos^2\theta=1\right]$$

= R.H.S.

Q. Prove that :—

$$\cos(\alpha) + \cos(\beta) + \cos(\gamma) + \cos(\alpha+\beta+\gamma)$$

$$= 4\cdot\cos\left(\frac{\alpha+\beta}{2}\right)\cdot\cos\left(\frac{\beta+\gamma}{2}\right)\cdot\cos\left(\frac{\gamma+\alpha}{2}\right)$$

Ans: L.H.S.

$$= \left(\cos(\alpha) + \cos(\beta)\right) + \left(\cos(\alpha+\beta+\gamma) + \cos(\gamma)\right)$$

$$= 2\cdot\cos\left(\frac{\alpha+\beta}{2}\right)\cdot\cos\left(\frac{\alpha-\beta}{2}\right) +$$

$$2\cdot\cos\left(\frac{\alpha+\beta+2\gamma}{2}\right)\cdot\cos\left(\frac{\alpha+\beta}{2}\right)$$

$$= 2\cdot\cos\left(\frac{\alpha+\beta}{2}\right)\cdot\left\{\cos\left(\frac{\alpha-\beta}{2}\right) + \cos\left(\frac{\alpha+\beta+2\gamma}{2}\right)\right\}$$

$$= 2\cdot\cos\left(\frac{\alpha+\beta}{2}\right)\left\{2\cdot\cos\left(\frac{\gamma+\alpha}{2}\right)\cdot\cos\left(\frac{\beta+\gamma}{2}\right)\right\}$$

$$= 4\cdot\cos\left(\frac{\alpha+\beta}{2}\right)\cdot 8\cos\left(\frac{\beta+\gamma}{2}\right)\cdot\cos\left(\frac{\beta+\gamma}{2}\right)$$

= R.H.S.

(P.T.O.)

Q. Prove that :—

$$\sin^2(6x) - \sin^2(4x) = \sin(10x)\cdot\sin(2x)$$

Ans:

L.H.S.

$= \sin^2(6x) - \sin^2(4x)$

$= \sin(6x+4x)\ \sin(6x-4x)$

$= \sin(10x)\cdot\sin(2x)$

Q. Prove that :—

$$\cos 20^\circ\cdot\cos 40^\circ\cdot\cos 60^\circ\cdot\cos 80^\circ = \frac{1}{16}.$$

Ans: L.H.S.

$= \cos 20^\circ\cdot\cos 40^\circ\cdot\cos 60^\circ\cdot\cos 80^\circ$

$= \frac{1}{2}\cos 60^\circ\cdot\cos 40^\circ \times (2\cdot\cos 80^\circ\cdot\cos 20^\circ)$

$= \frac{1}{2}\times\frac{1}{2}\times\cos 40^\circ\{\cos(80^\circ+20^\circ)+\cos(80^\circ-20^\circ)\}$

$= \frac{1}{4}\times\cos(40^\circ)\cdot(\cos(100^\circ)+\cos(60^\circ))$

$= \frac{1}{4}\times\cos(40^\circ)\left(\cos(100^\circ)+\frac{1}{2}\right)$

$= \frac{1}{4}\times\cos(40^\circ)\cdot\cos(100^\circ)+\frac{1}{8}\cdot\cos 40^\circ$

$= \frac{1}{8}\times(2\cos(40^\circ)\cdot\cos(100^\circ))+\frac{1}{8}\cdot\cos 40^\circ$

$= \frac{1}{8}\left[\cos(40^\circ+100^\circ)+\cos(100^\circ-40^\circ)\right]+\frac{1}{8}\cdot\cos 40^\circ$

$$= \frac{1}{8}\left[\cos(140^\circ) + \cos(60^\circ)\right] + \frac{1}{8}\cdot\cos(40^\circ)$$

$$= \frac{1}{8}\cdot\cos(140^\circ) + \frac{1}{8}\cdot\cos(60^\circ) + \frac{1}{8}\cdot\cos(40^\circ)$$

$$= \frac{1}{8}\cdot\cos(180^\circ - 40^\circ) + \frac{1}{16} + \frac{1}{8}\cdot\cos(40^\circ)$$

$$= \cancel{-\frac{1}{8}\cos(40^\circ)} + \frac{1}{16} + \cancel{\frac{1}{8}\cos(40^\circ)}$$

$$= \frac{1}{16} = R.H.S.$$

Q. Prove that :-

$$\sin 10^\circ \cdot \sin 50^\circ \cdot \sin 60^\circ \cdot \sin 70^\circ = \frac{\sqrt{3}}{16}$$

Ans: L.H.S.

$$= \frac{1}{2}\cdot\sin 60^\circ \sin 10^\circ\left(2\sin 70^\circ\cdot\sin 50^\circ\right)$$

$$= \frac{1}{2}\times\frac{\sqrt{3}}{2}\times\sin 10^\circ\left\{\cos(70-50) - \cos(120^\circ)\right\}$$

$$= \frac{\sqrt{3}}{4}\times\sin(10^\circ)\left\{\cos(20^\circ) - \cos(120^\circ)\right\}$$

$$= \frac{\sqrt{3}}{4}\times\sin(10^\circ)\left(\cos(20^\circ) + \frac{1}{2}\right) \quad \left\{\because \cos(120^\circ) = -\frac{1}{2}\right\}$$

$$= \frac{\sqrt{3}}{4}\times\sin(10^\circ)\cdot\cos(20^\circ) + \frac{\sqrt{3}}{8}\sin(10^\circ)$$

White Page

Date

Page

$= \frac{\sqrt{3}}{8} \cdot (2 \cdot \cos 20^\circ \cdot \sin 10^\circ) + \frac{\sqrt{3}}{8} \cdot \sin(10^\circ)$

$= \frac{\sqrt{3}}{8} [\sin(20^\circ + 10^\circ) - \sin(20^\circ - 10^\circ)] + \frac{\sqrt{3}}{8} \cdot \sin(10^\circ)$

$= \frac{\sqrt{3}}{8} \sin 30^\circ - \cancel{\frac{\sqrt{3}}{8} \cdot \sin(10^\circ)} + \cancel{\frac{\sqrt{3}}{8} \cdot \sin(10^\circ)}$

$= \left(\frac{\sqrt{3}}{8} \times \frac{1}{2}\right) = \frac{\sqrt{3}}{16} =$ R.H.S.

Q Prove that :-

$$\tan\left(\frac{\pi}{4} + x\right) = \frac{1 + \tan(x)}{1 - \tan(x)}$$

Ans :- L.H.S.

$= \tan\left(\frac{\pi}{4} + x\right)$

$= \frac{\tan\left(\frac{\pi}{4}\right) + \tan(x)}{1 - \tan\left(\frac{\pi}{4}\right) \cdot \tan(x)} = \frac{1 + \tan(x)}{1 - \tan(x)}$

= R.H.S.

Q Prove that :-

$$\tan\left(\frac{\pi}{4} - x\right) = \frac{1 - \tan(x)}{1 + \tan(x)}$$

Ans :- L.H.S.

$= \tan\left(\frac{\pi}{4} - x\right) = \frac{\tan\left(\frac{\pi}{4}\right) - \tan(x)}{1 + \tan\left(\frac{\pi}{4}\right) \cdot \tan(x)}$

$$= \frac{1-\tan(x)}{1+\tan(x)} = R.H.S.$$

Q Prove that :-

$$\frac{\sin(x)+\sin(3x)}{\cos(x)-\cos(3x)} = \cot(x)$$

Ans: L.H.S.

$$= \frac{\cancel{2}\sin\left(\frac{3x+x}{2}\right)\cdot\cos\left(\frac{3x-x}{2}\right)}{\cancel{2}\cdot\sin\left(\frac{3x+x}{2}\right)\cdot\sin\left(\frac{3x-x}{2}\right)}$$

$$= \frac{\cos\left(\frac{2x}{2}\right)}{\sin\left(\frac{2x}{2}\right)} = \frac{\cos(x)}{\sin(x)} = \cot(x)$$

$= R.H.S.$

Q. Prove that :-

$$\frac{\sin(7x)-\sin(5x)}{\cos(7x)+\cos(5x)} = \tan(x)$$

Ans: L.H.S.

$$= \frac{2.\cos\left(\frac{7x+5x}{2}\right)\cdot\sin\left(\frac{7x-5x}{2}\right)}{2.\cos\left(\frac{7x+5x}{2}\right)\cdot\cos\left(\frac{7x-5x}{2}\right)}$$

$$= \frac{\cancel{2}\cdot\cancel{\cos(6x)}\cdot\sin(x)}{\cancel{2}\cdot\cancel{\cos(6x)}\cdot\cos(x)} = \tan(x)$$

$= R.H.S.$

Q. Prove that :-

$$\frac{\sin(x) + \sin(3x) + \sin(5x)}{\cos(x) + \cos(3x) + \cos(5x)} = \tan(3x)$$

Ans: L.H.S.

$$= \frac{(\sin(5x) + \sin(x)) + \sin(3x)}{(\cos(5x) + \cos(x)) + \cos(3x)}$$

$$= \frac{2 \cdot \sin\left(\frac{5x+x}{2}\right) \cdot \cos\left(\frac{5x-x}{2}\right) + \sin(3x)}{2 \cdot \cos\left(\frac{5x+x}{2}\right) \cdot \cos\left(\frac{5x-x}{2}\right) + \cos(3x)}$$

$$= \frac{2 \cdot \sin(3x) \cdot \cos(2x) + \sin(3x)}{2 \cdot \cos(3x) \cdot \cos(2x) + \cos(3x)}$$

$$= \frac{\sin(3x)\,\cancel{[2 \cdot \cos(2x) + 1]}}{\cos(3x)\,\cancel{[2 \cdot \cos(2x) + 1]}}$$

$$= \tan(3x) = \text{R.H.S.}$$

Q. Prove that :-

$$\frac{(\sin(7x) + \sin(5x)) + (\sin(9x) + \sin(3x))}{(\cos(7x) + \cos(5x)) + (\cos(9x) + \cos(3x))} = \tan(6x)$$

Ans: L.H.S.

$$= \frac{2\cdot\sin\left(\frac{7x+5x}{2}\right)\cdot\cos\left(\frac{7x-5x}{2}\right)+2\cdot\sin\left(\frac{9x+3x}{2}\right)\cdot\cos\left(\frac{9x-3x}{2}\right)}{2\cdot\cos\left(\frac{7x+5x}{2}\right)\cdot\cos\left(\frac{7x-5x}{2}\right)+2\cdot\cos\left(\frac{9x+3x}{2}\right)\cdot\cos\left(\frac{9x-3x}{2}\right)}$$

$$= \frac{2\cdot\sin(6x)\cdot\cos(x)+2\cdot\sin(6x)\cdot\cos(3x)}{2\cdot\cos(6x)\cdot\cos(x)+2\cdot\cos(6x)\cdot\cos(3x)}$$

$$= \frac{\cancel{2}\cdot\sin(6x)\,\cancel{(\cos(x)+\cos(3x))}}{\cancel{2}\cdot\cos(6x)\,\cancel{(\cos(x)+\cos(3x))}}$$

$$= \frac{\sin(6x)}{\cos(6x)} = \tan(6x) = R.H.S.$$

Q. Prove that :—

$$\cot(4x)\cdot(\sin(5x)+\sin(3x)) = \cot(x)(\sin(5x)-\sin(3x))$$

Ans: L.H.S.

$$= \cot(4x)\cdot(\sin(5x)+\sin(3x))$$

$$= \cot(4x)\left(2\cdot\sin\left(\frac{5x+3x}{2}\right)\cdot\cos\left(\frac{5x-3x}{2}\right)\right)$$

$$= \cot(4x)(2\cdot\sin(4x)\cdot\cos(x))$$

$$= \frac{\cos(4x)}{\cancel{\sin(4x)}}\times 2\times\cancel{\sin(4x)}\times\cos(x)$$

$$= 2\cdot\cos(4x)\cdot\cos(x)$$

$$= R.H.S.$$

White Page

Date

Page

Q. Prove that :-

$(\sin(3x) + \sin(x)) \cdot \sin(x) + (\cos(3x) - \cos(x)) \cdot \cos(x)$
$= 0.$

Ans:
$$\because \sin C + \sin D = 2 \cdot \sin\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right)$$
$$\cos C - \cos D = 2 \sin\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{D-C}{2}\right)$$

L.H.S.

$$= \left(2 \cdot \sin\left(\frac{3x+x}{2}\right) \cdot \cos\left(\frac{3x-x}{2}\right)\right) \cdot \sin(x)$$
$$+ \left(2 \sin\left(\frac{3x+x}{2}\right) \cdot \sin\left(\frac{3x-x}{2}\right)\right) \cdot \cos(x)$$

$$= (2 \cdot \cancel{\sin(2x) \cdot \cos(x)}) \cdot \sin(x) -$$
$$(2 \cdot \sin(2x) \cdot \cancel{\sin(x)}) \cdot \cos(x)$$

$$= 0$$

Q. Prove that :-

$$(\cos(x) - \cos(y))^2 + (\sin(x) - \sin(y))^2$$
$$= 4 \cdot \sin^2\left(\frac{x-y}{2}\right)$$

(P.T.O.)

Ans: L.H.S.

$$= \left\{ \because \cos C - \cos D = 2\cdot\sin\left(\frac{C+D}{2}\right)\cdot\sin\left(\frac{D-C}{2}\right) \atop \sin C - \sin D = 2\cdot\cos\left(\frac{C+D}{2}\right)\cdot\sin\left(\frac{C-D}{2}\right) \right\}$$

$$= \left(-2\cdot\sin\left(\frac{x+y}{2}\right)\cdot\sin\left(\frac{x-y}{2}\right)\right)^2 + \left(2\cos\left(\frac{x+y}{2}\right)\cdot\sin\left(\frac{x-y}{2}\right)\right)^2$$

$$= 4\cdot\sin^2\left(\frac{x-y}{2}\right)\left[\cancel{\sin^2\left(\frac{x+y}{2}\right)+\cos^2\left(\frac{x+y}{2}\right)}^{\,1}\right]$$

$$= 4\cdot\sin^2\left(\frac{x-y}{2}\right) = R.H.S$$

proved

Q. Prove that :-

$$\frac{\sin(2x)-\sin(2y)}{\cos(2y)-\cos(2x)} = \cot(x+y)$$

Ans: L.H.S.

$$= \frac{2\cdot\cos\left(\frac{2x+2y}{2}\right)\cdot\sin\left(\frac{2x-2y}{2}\right)}{2\cdot\sin\left(\frac{2y+2x}{2}\right)\cdot\sin\left(\frac{2x-2y}{2}\right)}$$

$$= \frac{\cancel{2}\cos(x+y)\cdot\cancel{\sin(x-y)}}{\cancel{2}\cdot\sin(x+y)\cdot\cancel{\sin(x-y)}}$$

$$= \cot(x+y) = R.H.S.$$

Q. Prove that :-

$$\frac{\cos(x)+\cos(y)}{\cos(y)-\cos(x)} = \cot\left(\frac{x+y}{2}\right)\cdot\cot\left(\frac{x-y}{2}\right)$$

Ans: L.H.S.

$$= \frac{\cancel{2}\cdot\cos\left(\frac{x+y}{2}\right)\cdot\cos\left(\frac{x-y}{2}\right)}{\cancel{2}\cdot\sin\left(\frac{x+y}{2}\right)\cdot\sin\left(\frac{x-y}{2}\right)}$$

$$= \cot\left(\frac{x+y}{2}\right)\cdot\cot\left(\frac{x-y}{2}\right)$$

$$= R.H.S.$$

Q. Prove that :-

$$\frac{\sin(x)+\sin(y)}{\sin(x)-\sin(y)} = \tan\left(\frac{x+y}{2}\right)\cdot\cot\left(\frac{x-y}{2}\right)$$

(P.T.O.)

Ans: L.H.S.

$$= \frac{\sin(x) + \sin(y)}{\sin(x) - \sin(y)}$$

$$\left\{ \because \sin C + \sin D = 2 \cdot \sin\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right) \right.$$

$$\left. \sin C - \sin D = 2 \cdot \cos\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right) \right\}$$

$$= \frac{\cancel{2} \cdot \sin\left(\frac{x+y}{2}\right) \cdot \cos\left(\frac{x-y}{2}\right)}{\cancel{2} \cdot \cos\left(\frac{x+y}{2}\right) \cdot \sin\left(\frac{x-y}{2}\right)}$$

$$= \tan\left(\frac{x+y}{2}\right) \cdot \cot\left(\frac{x-y}{2}\right)$$

Q. $\sin(3x) + \sin(2x) - \sin(x) = 4 \cdot \sin(x) \cdot \cos\left(\frac{x}{2}\right) \cdot \cos\left(\frac{3x}{2}\right)$

Ans: L.H.S.

$$= (\sin(3x) - \sin(x)) + \sin(2x)$$

$$= 2 \cdot \cos\left(\frac{3x+x}{2}\right) \cdot \sin\left(\frac{3x-x}{2}\right) + \sin(2x)$$

$$= 2 \cdot \cos(2x) \cdot \sin(x) + \sin(2x)$$

$$= 2 \cdot \cos(2x) \cdot \sin(x) + 2\sin(x) \cdot \cos(x)$$

$$= 2 \cdot \sin(x) \left[\cos(2x) + \cos(x)\right]$$

$$= 2 \cdot \sin(x) \left[2\cos\left(\frac{2x+x}{2}\right) \cdot \cos\left(\frac{2x-x}{2}\right)\right]$$

$$= 4 \cdot \sin(x) \cdot \cos\left(\frac{3x}{2}\right) \cdot \cos\left(\frac{x}{2}\right) = \text{R.H.S}$$

Q. Prove that :-

$$\frac{\cos(4x)\cdot\sin(3x)-\cos(2x)\cdot\sin(x)}{\sin(4x)\cdot\sin(x)+\cos(6x)\cdot\cos(x)}=\tan(2x)$$

Ans: L.H.S.

$$=\frac{2\cdot\cos(4x)\cdot\sin(3x)-2\cdot\cos(2x)\cdot\sin(x)}{2\cdot\sin(4x)\cdot\sin(x)+2\cos(6x)\cdot\cos(x)}$$

$$=\frac{\sin(4x+3x)-\sin(4x-3x)-\{\sin(2x+x)-\sin(2x-x)\}}{\cos(4x-x)-\cos(4x+x)+\cos(6x+x)+\cos(6x-x)}$$

$$=\frac{\sin(7x)-\cancel{\sin(x)}-\sin(3x)+\cancel{\sin(x)}}{\cos(3x)-\cancel{\cos(5x)}+\cos(7x)+\cancel{\cos(5x)}}$$

$$=\frac{\sin(7x)-\sin(3x)}{\cos(3x)+\cos(7x)}$$

$$=\frac{\cancel{2}\cos\left(\cancel{\frac{7x+3x}{2}}\right)\cdot\sin\left(\frac{7x-3x}{2}\right)}{\cancel{2}\cdot\cos\left(\cancel{\frac{3x+7x}{2}}\right)\cdot\cos\left(\frac{3x-7x}{2}\right)}$$

$$=\frac{\sin(2x)}{\cos(2x)}=\tan(2x)$$

$=$ R.H.S.

White Page
Date
Page

Q. Prove that :—

$\sin 10^\circ \cdot \sin 30^\circ \cdot \sin 50^\circ \cdot \sin 70^\circ = \frac{1}{16}$.

Ans: L.H.S

$= \sin 10^\circ \cdot \sin 30^\circ \cdot \sin 50^\circ \cdot \sin 70^\circ$

$= \frac{1}{2}(2 \cdot \sin 70^\circ \cdot \sin 10^\circ) \cdot \sin 50^\circ \times \frac{1}{2}$

$= \frac{1}{4}\left[\cos(70^\circ - 10^\circ) - \cos(70^\circ + 10^\circ)\right] \sin 50^\circ$

$= \frac{1}{4}\left[\cos 60^\circ \cdot \sin 50^\circ - \cos 80^\circ \cdot \sin 50^\circ\right]$

$= \frac{1}{4}\left[\frac{1}{2} \cdot \sin 50^\circ - \cos 80^\circ \cdot \sin 50^\circ\right]$

$= \frac{1}{8}\left[\sin 50^\circ - 2 \cdot \cos 80^\circ \cdot \sin 50^\circ\right]$

$\left\{ \because 2 \cdot \cos A \cdot \sin B = \sin(A+B) - \sin(A-B) \right\}$

$= \frac{1}{8}\left[\sin 50^\circ - \sin(80 + 50) + \sin(80 - 50^\circ)\right]$

$= \frac{1}{8}\left[\sin 50^\circ - \sin(130^\circ) + \sin(30^\circ)\right]$

$= \frac{1}{8}\left[\sin 50^\circ - \sin(130^\circ) + \frac{1}{2}\right]$

$= \frac{1}{8}\left[\cancel{\sin 50^\circ} - \cancel{\sin(180^\circ - 50^\circ)} + \frac{1}{2}\right] = \frac{1}{16}$

$= R.H.S$

Q. Prove that :-

$$\sin 20^\circ \cdot \sin 40^\circ \cdot \sin 60^\circ \cdot \sin 80^\circ = \frac{3}{16}$$

Ans :

L.H.S

$\sin 20^\circ \cdot \sin 40^\circ \cdot \sin 60^\circ \cdot \sin 80^\circ$

$= \frac{1}{2}\left(2 \cdot \sin 80^\circ \cdot \sin 20^\circ\right) \cdot \sin 40^\circ \cdot \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{4}\left(2 \cdot \sin 80^\circ \cdot \sin 20^\circ\right) \cdot \sin 40^\circ$

$\left\{ \because 2 \sin A \cdot \sin B = \cos(A-B) - \cos(A+B) \right\}$

$= \frac{\sqrt{3}}{4}\left[\cos(80^\circ - 20^\circ) - \cos(80^\circ + 20^\circ)\right] \cdot \sin(40^\circ)$

$= \frac{\sqrt{3}}{4}\left[\cos(60^\circ) - \cos(100^\circ)\right] \cdot \sin(40^\circ)$

$= \frac{\sqrt{3}}{4}\left[\cos(60^\circ) \cdot \sin(40^\circ) - \cos(100^\circ) \cdot \sin(40^\circ)\right]$

$= \frac{\sqrt{3}}{4}\left[\frac{1}{2} \cdot \sin(40^\circ) - \cos(100^\circ) \cdot \sin(40^\circ)\right]$

$= \frac{\sqrt{3}}{8}\left[\sin(40^\circ) - 2\cos(100^\circ) \cdot \sin(40^\circ)\right]$

$= \frac{\sqrt{3}}{8}\left[(\sin 40^\circ) - \sin(140^\circ) + \sin(60^\circ)\right]$

White Page

Date

Page

$$= \frac{\sqrt{3}}{8}\left[\cancel{\sin 40^\circ} - \cancel{\sin(180^\circ - 40^\circ)} + \sin 60^\circ\right]$$

$$= \frac{\sqrt{3}}{8}\left[\frac{\sqrt{3}}{2}\right] = \frac{3}{16} = R.H.S.$$

Q. Prove that :—

$$\cos(10^\circ) \cdot \cos(30^\circ) \cdot \cos(50^\circ) \cdot \cos(70^\circ) = \frac{3}{16}.$$

Ans: L.H.S.

$$= \cos(10^\circ) \cdot \cos(30^\circ) \cdot \cos(50^\circ) \cdot \cos(70^\circ)$$

$$= \frac{1}{2}\left(2 \cdot \cos(70^\circ) \cdot \cos(10^\circ)\right) \cdot \cos(50^\circ) \cdot \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{4} \cdot \left[2 \cdot \cos(70^\circ) \cdot \cos(10^\circ)\right] \cdot \cos(50^\circ)$$

$$= \frac{\sqrt{3}}{4}\left[\cos(70^\circ + 10^\circ) + \cos(70^\circ - 10^\circ)\right] \cdot \cos(50^\circ)$$

$$= \frac{\sqrt{3}}{4}\left[\cos(80^\circ) + \cos(60^\circ)\right] \cdot \cos(50^\circ)$$

$$= \frac{\sqrt{3}}{4}\left[\cos(80^\circ) \cdot \cos(50^\circ) + \cos(60^\circ) \cdot \cos(50^\circ)\right]$$

$$= \frac{\sqrt{3}}{4}\left[\cos(80^\circ) \cdot \cos(50^\circ) + \frac{1}{2} \cdot \cos(50^\circ)\right]$$

$$= \frac{\sqrt{3}}{8}\left[2 \cdot \cos(80^\circ) \cdot \cos(50^\circ) + \cos(50^\circ)\right]$$

$$= \frac{\sqrt{3}}{8}\left[2\cdot\cos(80^\circ+50^\circ)\cdot\cos(80^\circ-50^\circ)+\cos 50^\circ\right]$$

$$= \frac{\sqrt{3}}{8}\left[2\cdot\cos(130^\circ)\cdot\cos(30^\circ)+\cos(50^\circ)\right]$$

$$= \frac{\sqrt{3}}{8}\left[\cancel{2}\cdot\cos(130^\circ)\times\frac{\sqrt{3}}{\cancel{2}}+\cos 50^\circ\right]$$

$$= \frac{\sqrt{3}}{8}\left[\cos(180-50^\circ)+\cos(30^\circ)+\cos(50^\circ)\right]$$

$$= \frac{\sqrt{3}}{8}\left[\cancel{-\cos(50^\circ)}+\cos(30^\circ)+\cancel{\cos(50^\circ)}\right]$$

$$= \frac{\sqrt{3}}{8}\times\frac{\sqrt{3}}{2} = \frac{3}{16} = \text{R.H.S.}$$

Q. If $\cos(x)+\cos(y)=\frac{1}{3}$ and $\sin(x)+\sin(y)=\frac{1}{4}$, prove that $\tan\left(\frac{x+y}{2}\right)=\frac{3}{4}$.

Ans:

$$\Rightarrow \frac{\sin(x)+\sin(y)}{\cos(x)+\cos(y)} = \frac{\frac{1}{4}}{\frac{1}{3}}$$

$$\Rightarrow \frac{\sin(x)+\sin(y)}{\cos(x)+\cos(y)} = \frac{3}{4}$$

$$\Rightarrow \frac{\cancel{2}\cdot \sin\left(\frac{x+y}{2}\right)\cdot \cancel{\cos\left(\frac{x-y}{2}\right)}}{\cancel{2}\cdot \cos\left(\frac{x+y}{2}\right)\cdot \cancel{\cos\left(\frac{x-y}{2}\right)}} = \frac{3}{4}$$

$$\therefore \tan\left(\frac{x+y}{2}\right) = \frac{3}{4} \quad \text{proved}$$

Q. If $\sin(x) = -\frac{1}{2}$ and $\pi < x < \frac{3\pi}{2}$, find the values of
(i) $\sin(2x)$ (ii) $\cos(2x)$ (iii) $\tan(2x)$

Ans: $\because$ x lies in Quadrant III rd, we have
$\sin(x) < 0$, $\cos(x) < 0$ & $\tan(x) > 0$.

Now,
$$\sin(x) = -\frac{1}{2} \quad (\text{given})$$

$$\therefore \cos(x) = -\sqrt{1-\sin^2(x)}$$

$$= -\sqrt{\left(1-\frac{1}{4}\right)} = -\sqrt{\frac{3}{4}} = -\frac{\sqrt{3}}{2}.$$

&

$$\tan(x) = \frac{\sin(x)}{\cos(x)} = \cancel{-}\frac{1}{\cancel{2}} \times \cancel{-}\frac{\cancel{2}}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

(i) $\sin(2x)$

$$= 2\cdot \sin(x)\cdot \cos(x)$$

$$= \cancel{2} \times \left(\cancel{-}\frac{1}{\cancel{2}}\right) \times \left(\cancel{-}\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\sqrt{3}}{2}$$

(ii.) $\cos(2x)$

$$= (2\cdot\cos^2(x) - 1)$$

$$= \left[2\times\left(-\frac{\sqrt{3}}{2}\right)^2 - 1\right]$$

$$= \left[\cancel{2}\times\frac{3}{\cancel{4}2} - 1\right]$$

$$= \frac{3}{2} - 1 = \frac{3-2}{2} = \frac{1}{2}.$$

(iii.) $\tan(2x)$

$$= \frac{\sin(2x)}{\cos(2x)} = \frac{\sqrt{3}}{\cancel{2}}\times\frac{\cancel{2}}{1} = \sqrt{3}$$

Q. If $\sin(x) = \frac{1}{3}$, find the value of $\sin(3x)$.

Ans: $\sin(3x) = 3\cdot\sin(x) - 4\cdot\sin^3(x)$

$$= \cancel{3}\cdot\left(\frac{1}{\cancel{3}}\right) - 4\cdot\left(\frac{1}{3}\right)^3$$

$$= 1 - \frac{4}{27} = \frac{27-4}{27} = \frac{23}{27}$$

Q. If $\cos(x) = \frac{1}{2}$, find the value of $\cos(3x)$.

Ans: $\cos(3x) = 4\cdot\cos^3(x) - 3\cdot\cos(x)$

$$= \left\{4\times\left(\frac{1}{2}\right)^3 - 3\times\frac{1}{2}\right\} = \left(\frac{1}{2} - \frac{3}{2}\right)$$

$$= -1.$$

Q. If $\cos(x) = \frac{4}{5}$ and x is acute, find the value of $\tan(2x)$.

Ans: $\cos(x) = \frac{4}{5} \Rightarrow \sec(x) = \frac{5}{4}$

$\Rightarrow \tan(x) = \sqrt{\sec^2(x) - 1} = \sqrt{\frac{25}{16} - 1}$

$= \sqrt{\frac{25-16}{16}} = \sqrt{\frac{9}{16}} = \frac{3}{4}$.

$\therefore \tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)} = \frac{2 \times \frac{3}{4}}{1 - \frac{9}{16}}$

$\Rightarrow \frac{\frac{6}{4}}{\frac{7}{16}} = \frac{\cancel{6}^{3}}{\cancel{4}_{2}} \times \frac{\cancel{16}^{8}}{7} = \frac{24}{7}$.

Q. If $\tan(x) = \frac{1}{7}$ and $\tan(y) = \frac{1}{3}$, show that $\cos(2x) = \sin(4y)$.

Ans:

$\cos(2x) = \frac{1 - \tan^2(x)}{1 + \tan^2(x)} = \frac{\left(1 - \frac{1}{49}\right)}{\left(1 + \frac{1}{49}\right)}$

$\Rightarrow \frac{49-1}{49+1} = \frac{\cancel{48}\,24}{\cancel{50}\,25} = \frac{24}{25}$

(P.T.O.)

$$\sin(4y) = 2\cdot\sin(2y)\cdot\cos(2y)$$

$$= 2\cdot \times \frac{2\tan(y)}{(1+\tan^2(y))} \times \frac{1-\tan^2(y)}{1+\tan^2(y)}$$

$$= 2\times \frac{\left(2\times\frac{1}{3}\right)}{1+\left(\frac{1}{3}\right)^2} \times \frac{1-\left(\frac{1}{3}\right)^2}{1+\left(\frac{1}{3}\right)^2}$$

$$= \left(\frac{4}{3}\times\frac{9}{10}\times\frac{8}{9}\times\frac{9}{10}\right) = \frac{24}{25}$$

$$\therefore \cos(2x) = \sin(4y)$$

Q. Prove that :-

$$\sin\left(\frac{\pi}{6}\right)\cdot\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{4}$$

Ans: L.H.S

$$= \frac{1}{2}\times\left(2\cdot\sin\left(\frac{\pi}{6}\right)\cdot\cos\left(\frac{\pi}{6}\right)\right)$$

$$= \frac{1}{2}\times\sin\left(\cancel{2}\times\frac{\pi}{\cancel{6}_3}\right)$$

$\left\{\because 2\sin(x)\cdot\cos(x) = \sin(2x)\right\}$

$$= \frac{1}{2}\times\sin\left(\frac{\pi}{3}\right)$$

$$= \frac{1}{2}\times\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} = R.H.S$$

Q. Prove that :—

$$\cos^2\left(\frac{\pi}{12}\right) - \sin^2\left(\frac{\pi}{12}\right)$$

Ans:

$$\left\{\because \cos^2(x) - \sin^2(x) = \cos(2x)\right\}$$

$$\Rightarrow \cos\left(\cancel{2}\times\frac{\pi}{\cancel{12}_6}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

Q. Prove that :—

$$\frac{\sin(2x)}{1-\cos(2x)} = \cot(x)$$

Ans: L.H.S.

$$\Rightarrow \frac{\sin(2x)}{1-\cos(2x)} = \frac{\cancel{2}\cdot\sin\cancel{(x)}\cdot\cos(x)}{\cancel{2}\cdot\sin^{\cancel{2}}(x)}$$

$$\left\{\begin{aligned}\because \quad \sin(2x) &= 2\cdot\sin(x)\cdot\cos(x) \\ 1-\cos(2x) &= 2\cdot\sin^2(x)\end{aligned}\right\}$$

$$= \frac{\cos(x)}{\sin(x)} = \cot(x) = R.H.S$$

proved

Q. Prove that :—

$$\frac{1-\cos(2\cdot x)}{1+\cos(2\cdot x)} = \tan^2(x)$$

(P.T.O.)

Ans: L.H.S

$$= \frac{1-\cos(2x)}{1+\cos(2x)} \quad \left\{ \begin{array}{l} \because \\ 1-\cos(2x) = 2\cdot\sin^2(x) \\ 1+\cos(2x) = 2\cdot\cos^2(x) \end{array} \right\}$$

$$= \frac{\cancel{2}\cdot\sin^2(x)}{\cancel{2}\cdot\cos^2(x)} = \tan^2(x)$$

$= R.H.S$ proved //

Q. Prove that :-

$$\cos(4x) = 1 - 8\cdot\sin^2(x)\cdot\cos^2(x)$$

Ans: L.H.S

$= \cos(4x)$

$= \cos 2(2x)$

let $2x = \theta$

$= \cos 2\theta$

$= \cos^2(\theta) - \sin^2(\theta)$

$= (1 - 2\cdot\sin^2(\theta))$ $\left\{ \because \cos^2(\theta) = 1 - \sin^2(\theta) \right\}$

putting $\theta = 2x$; we get

$= \{1 - 2\,\sin(2x)^2\}$

$= 1 - 2(2\cdot\sin(x)\cdot\cos(x))^2$

$= 1 - 2\times 4\cdot\sin^2(x)\cdot\cos^2(x)$

$= 1 - 8\cdot\sin^2(x)\cdot\cos^2(x)$

$= R.H.S$

proved

White Page

Date

Page

Q. Prove that :-

$$\frac{1+\sin(2x)-\cos(2x)}{1+\sin(2x)+\cos(2x)} = \tan(x)$$

Ans:

L.H.S.

$$= \frac{1+\sin(2\cdot x)-\cos(2\cdot x)}{1+\sin(2\cdot x)+\cos(2\cdot x)}$$

$$= \frac{(1-\cos(2x))+\sin(2x)}{(1+\cos(2x))+\sin(2x)}$$

$$= \frac{2\cdot\sin^2(x)+2\cdot\sin(x)\cdot\cos(x)}{2\cdot\cos^2(x)+2\cdot\sin(x)\cdot\cos(x)}$$

$$= \frac{\cancel{2}\sin(x)\left[\cancel{\sin(x)+\cos(x)}\right]}{\cancel{2}\cdot\cos(x)\left[\cancel{\cos(x)+\sin(x)}\right]}$$

$$= \tan(x) = R.H.S.$$

Q. Prove that :-

$$\frac{1-\sin(2x)}{1+\sin(2x)} = \tan^2\left(\frac{\pi}{4}-x\right)$$

(P.T.O.)

White Page
Date
Page

Ans: L.H.S.

$$= \frac{1-\sin(2x)}{1+\sin(2x)} \quad \left\{\because \sin(2x) = \cos\left(\frac{\pi}{2}-2x\right)\right\}$$

$$= \frac{1-\cos\left(\frac{\pi}{2}-2x\right)}{1+\cos\left(\frac{\pi}{2}-2x\right)}$$

$$\left\{\begin{aligned} \because 1-\cos\theta &= 2\cdot\sin^2(\theta) \\ 1+\cos\theta &= 2\cdot\cos^2(\theta) \end{aligned}\right\}$$

$$= \frac{\cancel{2}\cdot\sin^2\cancel{(\theta)}(2x)}{\cancel{2}\cdot\cos^2\theta\cancel{(2x)}}$$

$$= \frac{\cancel{2}\cdot\sin^2\left(\frac{\pi}{4}-x\right)}{\cancel{2}\cdot\cos^2\left(\frac{\pi}{4}-x\right)}$$

$$= \tan^2\left(\frac{\pi}{4}-x\right) = \text{R.H.S.}$$ proved

Q. Prove that :-

$$\frac{\cos(x)+\sin(x)}{\cos(x)-\sin(x)} - \frac{\cos(x)-\sin(x)}{\cos(x)+\sin(x)} = 2\cdot\tan(2x)$$

(P.T.O.)

Ans: L.H.S.

$$= \frac{\cos(x)+\sin(x)}{\cos(x)-\sin(x)} - \frac{\cos(x)-\sin(x)}{\cos(x)+\sin(x)}$$

$$= \frac{(\cos(x)+\sin(x))^2 - (\cos(x)-\sin(x))^2}{\cos^2(x)-\sin^2(x)}$$

$$= \frac{\cancel{\cos^2(x)}+\cancel{\sin^2(x)}+2\sin(x)\cos(x)-\cancel{\cos^2(x)}-\cancel{\sin^2(x)} + 2\sin(x)\cdot\cos(x)}{\cos^2(x)-\sin^2(x)}$$

$$= \frac{4\cdot\sin(x)\cdot\cos(x)}{\cos^2(x)-\sin^2(x)} = \frac{2\cdot\sin(2x)}{\cos(2x)}$$

$$= 2\cdot\tan(2x) = \text{R.H.S.}$$

Q. Prove that :-

$$\tan(4\theta) = \frac{4\cdot\tan(\theta)\,(1-\tan^2(\theta))}{1-6\cdot\tan^2(\theta)+\tan^4(\theta)}$$

Ans: L.H.S.

$= \tan(4\theta)$

$= \tan 2(2\theta)$

$= \tan 2\cdot x$, where $x = 2\cdot\theta$

$$= \frac{2\tan(x)}{1-\tan^2(x)} = \frac{2\cdot\tan(2\theta)}{1-\tan^2(2\theta)}$$

$$= \frac{2\times\frac{2\tan\theta}{1-\tan^2\theta}}{1-\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)^2} = \frac{\frac{4\tan\theta}{1-\tan^2\theta}}{\frac{(1-\tan^2\theta)^2-(2\tan\theta)^2}{(1-\tan^2\theta)^2}}$$

$$= \frac{4\cdot\tan(\theta)\cdot\left(1-\tan^2(\theta)\right)^{\cancel{2}}}{\cancel{(1-\tan^2\theta)}\left\{(1-\tan^2\theta)^2-(2\tan\theta)^2\right\}}$$

$$= \frac{4\cdot\tan(\theta)\,(1-\tan^2\theta)}{(1-\tan^2(\theta))^2-4\cdot\tan^2(\theta)}$$

$$= \frac{4\cdot\tan(\theta)\cdot(1-\tan^2(\theta))}{1+\tan^4\theta-2\tan^2\theta-4\tan^2\theta}$$

$$= \frac{4\cdot\tan(\theta)\cdot(1-\tan^2(\theta))}{1+\tan^4(\theta)-6\cdot\tan^2(\theta)}$$

$= R.H.S.$

Q. Prove that :-

$$\frac{\tan(5\theta)+\tan(3\theta)}{\tan(5\theta)-\tan(3\theta)} = 4\cdot\cos(2\theta)\cdot\cos(4\theta)$$

Ans: L.H.S.

$$= \frac{\tan(5\theta)+\tan(3\theta)}{\tan(5\theta)-\tan(3\theta)}$$

$$= \frac{\left(\frac{\sin(5\theta)}{\cos(5\theta)}\right)+\left(\frac{\sin(3\theta)}{\cos(3\theta)}\right)}{\left(\frac{\sin(5\theta)}{\cos(5\theta)}\right)-\left(\frac{\sin(3\theta)}{\cos(3\theta)}\right)}$$

$$= \frac{\dfrac{\sin(5\theta)\cdot\cos(3\theta) + \sin(3\theta)\cdot\cos(5\theta)}{\cancel{\cos(5\theta)\cdot\cos(3\theta)}}}{\dfrac{\sin(5\theta)\cdot\cos(3\theta) - \sin(3\theta)\cdot\cos(5\theta)}{\cancel{\cos(5\theta)\cdot\cos(3\theta)}}}$$

$$= \frac{\sin(5\theta)\cdot\cos(3\theta) + \sin(3\theta)\cdot\cos(5\theta)}{\sin(5\theta)\cdot\cos(3\theta) - \sin(3\theta)\cdot\cos(5\theta)}$$

$$\left\{ \begin{array}{l} \because \sin A\cdot\cos B + \cos A\cdot\sin B = \sin(A+B) \\ \sin A\cdot\cos B - \cos A\cdot\sin B = \sin(A-B) \end{array} \right\}$$

$$= \frac{\sin(5\theta+3\theta)}{\sin(5\theta-3\theta)} = \frac{\sin(8\theta)}{\sin(2\theta)}$$

$$= \frac{2\cdot\sin(4\theta)\cdot\cos(4\theta)}{\sin(2\theta)}$$

$$= \frac{2\cdot 2\cdot\cancel{\sin(2\theta)}\cdot\cos(2\theta)\cdot\cos(4\theta)}{\cancel{\sin(2\theta)}}$$

$$= 4\cdot\cos(2\theta)\cdot\cos(4\theta)$$

$= R.H.S$

(P.T.O.)

Q. Prove that :—

$$\frac{\sec(8\theta)-1}{\sec(4\theta)-1} = \frac{\tan(8\theta)}{\tan(2\theta)}$$

Ans:

L.H.S.

$$= \frac{\sec(8\theta)-1}{\sec(4\theta)-1}$$

$$= \frac{\left(\frac{1}{\cos(8\theta)}-1\right)}{\left(\frac{1}{\cos(4\theta)}-1\right)} = \frac{(1-\cos 8\theta)}{(1-\cos 4\theta)} \cdot \frac{\cos(4\theta)}{\cos(8\theta)}$$

$$= \frac{2\cdot\sin^2(4\theta)\cdot\cos(4\theta)}{2\cdot\sin^2(2\theta)\cdot\cos(8\theta)}$$

$$\left[\begin{array}{l} \because 1-\cos 2\theta \\ = 2\cdot\sin^2\theta \\ 1+\cos 2\theta \\ = 2\cos^2(\theta) \end{array}\right]$$

$$= \frac{(2\cdot\sin(4\theta)\cdot\cos(4\theta))\cdot\sin(4\theta)}{(2\cdot\sin^2(2\theta))\cdot\cos(8\theta)}$$

$$= \frac{\sin(8\theta)\cdot\cancel{2}\sin\cancel{(2\theta)}\cdot\cos(2\theta)}{\cos(8\theta)\cdot(\cancel{2}\cdot\sin^{\cancel{2}}(2\theta))}$$

$$= \cancel{\cot 8\theta}\ \tan(8\theta)\cdot\cot(2\theta)$$

$$= \frac{\tan(8\theta)}{\tan(2\theta)} = \text{R.H.S.}$$

Q. Show that :—

$$\sqrt{2+\sqrt{2+2\cdot\cos(4\theta)}} = 2\cdot\cos(\theta).$$

Ans: L.H.S.

$$= \sqrt{2+\sqrt{2+2\cdot\cos(4\theta)}}$$

$$= \sqrt{2+\sqrt{2(1+\cos 4\theta)}}$$

$$= \sqrt{2+\sqrt{2\cdot 2\cdot\cos^2(2\theta)}}$$

$$= \sqrt{2+\sqrt{4\cdot\cos^2(2\theta)}} \quad \left\{ \because 1+\cos(2\theta) = 2\cdot\cos^2(\theta) \right\}$$

$$= \sqrt{2+2\cdot\cos(2\theta)}$$

$$= \sqrt{2(1+\cos(2\theta))}$$

$$= \sqrt{2\cdot 2\cdot\cos^2(\theta)}$$

$$= \sqrt{4\cdot\cos^2(\theta)}$$

$$= 2\cdot\cos(\theta)$$

Q. Show that :—

$$\cos(5\cdot x) = 16\cdot\cos^5(x) - 20\cdot\cos^3(x) + 5\cdot\cos(x)$$

Ans: L.H.S.

$$= \cos(5x)$$

$$= \cos(3x+2x)$$

$$= \cos(3x)\cdot\cos(2x) - \sin(3x)\cdot\sin(2x)$$

$$= (4\cdot\cos^3(x) - 3\cos(x))(2\cdot\cos^2(x) - 1) - (3\cdot\sin(x) - 4\cdot\sin^3(x))\cdot(2\cdot\sin(x)\cdot\cos(x))$$

$$= \left(8\cdot\cos^5(x) - 10\cdot\cos^3(x) + 3\cdot\cos(x)\right) - \left(6\cdot\sin^2(x)\cdot\cos(x)\right) + 8\cdot\sin^4(x)\cdot\cos(x)$$

$$= 8\cdot\cos^5(x) - 10\cdot\cos^3(x) + 3\cdot\cos(x) - 6\left(1-\cos^2(x)\right)\cdot\cos(x) + 8\left(1-\cos^2(x)\right)^2\cdot\cos(x)$$

$$= 16\cdot\cos^5(x) - 20\cdot\cos^3(x) + 5\cdot\cos(x)$$

= R.H.S. proved

Q. Prove that :—

$$\cos(6x) = 32\cdot\cos^6(x) - 48\cdot\cos^4(x) + 18\cdot\cos^2(x) - 1.$$

Ans:

L.H.S.

$$= \cos(6x)$$
$$= \cos 2(3x)$$
$$= \cos 2\theta \text{, where } 3x = \theta$$
$$= 2\cos^2\theta - 1$$
$$= 2\cdot\cos^2(3x) - 1.$$
$$= 2\left(4\cdot\cos^3(x) - 3\cos(x)\right)^2 - 1$$
$$= 2\left(16\cos^6(x) + 9\cos^2(x) - 24\cos^4(x)\right) - 1$$
$$= 32\cdot\cos^6(x) - 48\cdot\cos^4(x) + 18\cdot\cos^2(x) - 1$$

= R.H.S. proved

Q Prove that :—

$$\cot(2x)\cdot\cot(x) - \cot(3x)\cdot\cot(2x) - \cot(3x)\cdot\cot(x) = 1.$$

~~Ans: 10/4/50~~

$\Rightarrow \cot(3x) = \cot(2x + x)$

$\Rightarrow \cot(3x) = \dfrac{\cot(2x)\cdot\cot(x) - 1}{\cot(2x) + \cot(x)}$

$\Rightarrow \cot(3x)\cdot\cot(2x) + \cot(3x)\cdot\cot(x) =$

$\cot(2x)\cdot\cot(x) - 1$

$\Rightarrow \cot(2x)\cdot\cot(x) - \cot(3x)\cdot\cot(2x) - \cot(3x)\cdot\cot(x)$
$= 1$

proved

★★★ Find the value of :—

(i) $\sin(18^\circ)$

let $\theta = 18^\circ$, Then

$\theta = 18^\circ \Rightarrow 5\theta = 90^\circ$

$\Rightarrow 2\theta = (90^\circ - 3\theta)$

$\Rightarrow \sin(2\theta) = \sin(90^\circ - 3\theta)$

$\Rightarrow 2\cdot\sin\theta\cdot\cos\theta = \cos(3\theta)$

$\Rightarrow 2\cdot\sin\theta\cdot\cos\theta = 4\cdot\cos^3(\theta) - 3\cdot\cos(\theta)$

$\Rightarrow 2\cdot\sin\theta\cdot\cos\theta - 4\cdot\cos^3(\theta) + 3\cdot\cos(\theta) = 0$

$\Rightarrow \cos(\theta)\,(2\cdot\sin(\theta) - 4\cdot\cos^2(\theta) + 3) = 0$

$\cos\theta = \cos 18^\circ \neq 0$ क्योंकी 0 से 45° में cos का value 1 से $1/\sqrt{2}$ के बीच में रहता है ; इसलिए इसको discard कर रहे हैं।

$\Rightarrow 2\cdot\sin(\theta) - 4\cdot\cos^2(\theta) + 3 = 0$

$\Rightarrow 2\cdot\sin(\theta) - 4(1-\sin^2(\theta)) + 3 = 0$

$\Rightarrow 2\sin(\theta) - 4 + 4\sin^2(\theta) + 3 = 0$

$\Rightarrow 4\sin^2(\theta) + 2\sin(\theta) - 1 = 0$

$$\sin(\theta) = \frac{-2 \pm \sqrt{4+16}}{8}$$

$$= \frac{-1 \pm \sqrt{5}}{4}$$

$\frac{-1-\sqrt{5}}{4}$ ✗ $\qquad \frac{\sqrt{5}-1}{4}$ ✓

क्यों ?

हमे बखुबी पता है की sin 0 से 45° मे value 0 से $1/\sqrt{2}$ के बीच मे होता है इसलिए ये वाला आंसर जो की finally negative quantity है इसलिए उसको consider नही करेंगे।

$$\boxed{\therefore \sin(18^\circ) = \left(\frac{\sqrt{5}-1}{4}\right)}$$

(ii) $\cos(18^\circ)$

$$\because \sin^2\theta + \cos^2\theta = 1$$
$$\cos^2\theta = 1 - \sin^2(\theta)$$

$$\Rightarrow \cos^2(\theta) = 1 - \sin^2(\theta)$$
$$\Rightarrow \cos^2(18^\circ) = 1 - \sin^2(18^\circ$$
$$\Rightarrow \cos^2 18^\circ = 1 - \frac{(\sqrt{5}-1)^2}{(4)^2}$$
$$\Rightarrow \cos^2 18^\circ = \frac{16 - (5+1-2\sqrt{5})}{16}$$
$$\Rightarrow \cos^2 18^\circ = \frac{16 - 6 + 2\sqrt{5}}{16}$$
$$\Rightarrow \cos^2 18^\circ = \frac{10+2\sqrt{5}}{16}$$
$$\Rightarrow \cos(18^\circ) = \frac{\sqrt{10+2\sqrt{5}}}{4}$$

-ve वाला answer इसलिए neglect किए क्योंकी cos 0 से 45° में उसकी value 1 से 1/√2 के बीच में होती है।

$$\therefore \boxed{\cos(18^\circ) = \frac{\sqrt{10+2\sqrt{5}}}{4}}$$

(iii) $\cos(36^\circ)$

$= \cos(36^\circ)$

$= \cos 2(18^\circ)$

$= 1 - 2\cdot\sin^2(18^\circ)$

$\left\{\because \cos(2\theta) = 1 - 2\cdot\sin^2(\theta)\right\}$

$= \left\{1 - 2\cdot\dfrac{(\sqrt{5}-1)^2}{(4)^2}\right\}$

$= \left\{1 - \cancel{2}\cdot\dfrac{(\sqrt{5}-1)^2}{\cancel{16}\ 8}\right\}$

$= \left\{1 - \dfrac{(\sqrt{5}-1)^2}{8}\right\}$

$= \left\{\dfrac{8-(5+1-2\sqrt{5})}{8}\right\}$

$= \left\{\dfrac{8-6+2\sqrt{5}}{8}\right\}$

$= \left\{\dfrac{2+2\sqrt{5}}{8}\right\} = \dfrac{\sqrt{5}+1}{4}$.

$$\boxed{\therefore \cos(36^\circ) = \frac{\sqrt{5}+1}{4}}$$

(iv) $\sin(36^\circ)$

$= \cancel{\sin 2(18^\circ)}$

$$\left\{ \begin{aligned} &\because \sin^2(\theta) + \cos^2(\theta) = 1 \\ &\sin^2\theta = 1 - \cos^2\theta \\ &\sin\theta = \sqrt{1 - \cos^2(\theta)} \end{aligned} \right\}$$

$$\Rightarrow \sin(36^\circ) = \sqrt{1 - \cos^2 36^\circ}$$

$$\Rightarrow \sin(36^\circ) = \left\{1 - \frac{(\sqrt{5}+1)^2}{16}\right\}^{1/2}$$

$$\Rightarrow \sin(36^\circ) = \left\{\frac{16 - 5 - 1 - 2\sqrt{5}}{16}\right\}^{1/2}$$

$$\Rightarrow \sin(36^\circ) = \left\{\frac{10 - 2\sqrt{5}}{16}\right\}^{1/2}$$

$$\Rightarrow \sin(36^\circ) = \frac{\sqrt{10 - 2\sqrt{5}}}{4}.$$

$$\therefore \boxed{\sin(36^\circ) = \frac{\sqrt{10-2\sqrt{5}}}{4}}$$

(v) $\sin(72^\circ)$

Ans:
$$= \sin(90^\circ - 18^\circ)$$
$$= \cos(18^\circ)$$
$$= \frac{\sqrt{10+2\sqrt{5}}}{4}$$

$$\therefore \boxed{\sin(72^\circ) = \frac{\sqrt{10+2\sqrt{5}}}{4}}$$

(vi) $\cos(72^\circ)$

Ans:
$$= \cos(90^\circ - 18^\circ)$$
$$= \sin 18^\circ$$
$$= \frac{(\sqrt{5}-1)}{4}$$

$$\therefore \boxed{\cos(72^\circ) = \frac{\sqrt{5}-1}{4}}$$

(P.T.O.)

White Page
Date
Page

(vii) $\sin(54^\circ)$

$= \sin(90^\circ - 36^\circ)$
$= \cos(36^\circ)$
$= \frac{\sqrt{5}+1}{4}$

$\boxed{\therefore \sin(54^\circ) = \frac{\sqrt{5}+1}{4}}$

(viii) $\cos(54^\circ)$

$= \cos(90^\circ - 36^\circ)$
$= \sin(36^\circ)$
$= \frac{\sqrt{10-2\sqrt{5}}}{4}$

$\boxed{\therefore \cos(54^\circ) = \frac{\sqrt{10-2\cdot\sqrt{5}}}{4}}$

Q. Prove that :—

$$\sin\left(\frac{\pi}{10}\right) + \sin\left(\frac{13\pi}{10}\right) = -\frac{1}{2}.$$

Ans: L.H.S.

$= \sin\left(\frac{\pi}{10}\right) + \sin\left(\frac{13\pi}{10}\right)$

$= \sin\left(\frac{\pi}{10}\right) + \sin\left(\pi + \frac{3\pi}{10}\right)$

$= \sin\left(\frac{\pi}{10}\right) - \sin\left(\frac{3\pi}{10}\right)$ $\left\{\because \sin(\pi-\theta) = -\sin\theta\right\}$

$= \sin 18^\circ - \sin 54^\circ$
$= \sin 18^\circ - \sin(90^\circ - 36^\circ)$
$= \sin 18^\circ - \cos 36^\circ$

$$= \left\{ \frac{(\sqrt{5}-1)}{4} - \frac{(\sqrt{5}+1)}{4} \right\}$$

$$= \frac{\sqrt{5}-1-\sqrt{5}-1}{4}$$

$$= \frac{-\cancel{2}}{\cancel{4}\,2} = -\frac{1}{2} = \text{R.H.S.}$$

Q. Prove that :-

$$\cos^2(48^\circ) - \sin^2(12^\circ) = \frac{(\sqrt{5}+1)}{8}$$

Ans:- L.H.S.

$$= \cos^2(48^\circ) - \sin^2(12^\circ)$$

$$= \frac{1}{2}\cdot\left(2\cdot\cos^2(48^\circ) - 2\cdot\sin^2(12^\circ)\right)$$

$$= \frac{1}{2}\left\{(1+\cos 96^\circ) - (1-\cos 24^\circ)\right\}$$

$$= \frac{1}{2}\cdot\left\{\cos(96^\circ) + \cos(24^\circ)\right\}$$

$$\left\{ \begin{array}{l} \because\ 1+\cos(2\theta) = 2\cdot\cos^2(\theta) \\ 1-\cos(2\theta) = 2\cdot\sin^2(\theta) \\ \cos C + \cos D = 2\cdot\cos\left(\frac{C+D}{2}\right)\cdot\cos\left(\frac{C-D}{2}\right) \end{array} \right\}$$

$$= \frac{1}{2}\left\{2\cdot\cos\left(\frac{96^\circ+24^\circ}{2}\right)\cdot\cos\left(\frac{96^\circ-24^\circ}{2}\right)\right\}$$

$= \frac{1}{2} \times 2 \cdot \cos(60^\circ) \cdot \cos(36^\circ)$

$= \frac{1}{\cancel{2}} \times \cancel{2} \times \frac{1}{2} \times \frac{(\sqrt{5}+1)}{4}$

$= \frac{\sqrt{5}+1}{8} = R.H.S.$ ~~proved~~

Q. Prove that :—

$\sin^2(72^\circ) - \sin^2(60^\circ) = \frac{(\sqrt{5}-1)}{8}$

Ans: L.H.S.

$= \sin^2(72^\circ) - \sin^2(60^\circ)$

$= \frac{1}{2}\left(2\cdot\sin^2(72^\circ) - 2\cdot\sin^2(60^\circ)\right)$

$= \frac{1}{2}\left\{\left(1-\cos(144^\circ)\right) - \left(1-\cos 120^\circ\right)\right\}$

$\left\{\because 2\sin^2(\theta) = 1-\cos(2\theta)\right\}$

$= \frac{1}{2}\left(\cos(120^\circ) - \cos(144^\circ)\right)$

$= \frac{1}{2}\left(-\frac{1}{2} - \cos(144^\circ)\right)$ $\left[\because \cos(120^\circ) = -\frac{1}{2}\right]$

$= -\frac{1}{4} - \frac{1}{2}\cdot\cos(144^\circ)$

$= -\frac{1}{4} - \frac{1}{2}\cdot\cos(180^\circ - 36^\circ)$

$= -\frac{1}{4} + \frac{1}{2}\cdot\cos(36^\circ)$

$$= -\frac{1}{4} + \frac{1}{2}\left(\frac{\sqrt{5}+1}{4}\right)$$

$$\left\{\because \cos 36^\circ = \frac{(\sqrt{5}+1)}{4}\right\}$$

$$= \frac{(\sqrt{5}-1)}{8} = \text{R.H.S}$$ proved//

Q. Prove that :—

$$\cos 6^\circ \cdot \cos 42^\circ \cdot \cos 66^\circ \cdot \cos 78^\circ = \frac{1}{16}.$$

Ans :

L.H.S

$$= \cos 6^\circ \cdot \cos 42^\circ \cdot \cos 66^\circ \cdot \cos 78^\circ$$

$$= \frac{1}{2}(2\cdot\cos 66^\circ \cdot \cos 6^\circ)\cdot\frac{1}{2}(2\cdot\cos 78^\circ\cdot\cos 42^\circ)$$

$$= \frac{1}{4}\left[\cos(66^\circ+6^\circ)+\cos(66^\circ-6^\circ)\right] \times \left[\cos(78^\circ+42^\circ)+\cos(78^\circ-42^\circ)\right].$$

$$= \frac{1}{4}\cdot\left(\cos(72^\circ)+\cos(60^\circ)\right)\cdot\left(\cos(120^\circ)+\cos(36^\circ)\right)$$

$$= \frac{1}{4}\cdot\left(\sin 18^\circ + \frac{1}{2}\right)\cdot\left(-\frac{1}{2}+\cos(36^\circ)\right)$$

$$\left\{\because \cos(72^\circ) = \cos(90^\circ-18^\circ) = \sin 18^\circ\right\}$$

$$\left\{ \because \sin(18^\circ) = \frac{(\sqrt{5}-1)}{4} \ \& \ \cos(36^\circ) = \frac{(\sqrt{5}+1)}{4} \right\}$$

$$= \frac{1}{4}\left[\frac{(\sqrt{5}-1)}{4} + \frac{1}{2}\right]\left[-\frac{1}{2} + \frac{\sqrt{5}+1}{4}\right]$$

$$= \frac{1}{4} \times \frac{\sqrt{5}+1}{4} \times \frac{\sqrt{5}-1}{4}$$

$$= \frac{5-1}{64} = \frac{\cancel{4}}{\cancel{64}} = \frac{1}{16} = R.H.S.$$ proved //

Q. Prove that :—

$$\sin\left(\frac{\pi}{5}\right) \cdot \sin\left(\frac{2\pi}{5}\right) \cdot \sin\left(\frac{3\pi}{5}\right) \cdot \sin\left(\frac{4\pi}{5}\right) = \frac{5}{16}.$$

Ans: L.H.S.

$$= \sin\left(\frac{\pi}{5}\right) \cdot \sin\left(\frac{2\pi}{5}\right) \cdot \sin\left(\frac{3\pi}{5}\right) \cdot \sin\left(\frac{4\pi}{5}\right)$$

$$= \sin\left(\frac{\pi}{5}\right) \cdot \sin\left(\frac{2\pi}{5}\right) \cdot \sin\left(\pi - \frac{2\pi}{5}\right) \cdot \sin\left(\pi - \frac{\pi}{5}\right)$$

$$= \sin^2\left(\frac{\pi}{5}\right) \cdot \sin^2\left(\frac{2\pi}{5}\right)$$

$$= \sin^2(36^\circ) \times \sin^2(72^\circ)$$

$$\left\{ \because \sin(72^\circ) = \sin(90^\circ - 18^\circ) = \cos 18^\circ \right\}$$

$$= (\sin(36^\circ))^2 \times (\cos(18^\circ))^2$$

$$= \frac{(10 - 2\sqrt{5})}{16} \times \frac{(10 + 2\sqrt{5})}{16} = \frac{100 - 20}{16 \times 16}$$

$$= \frac{80}{16 \times 16} = \frac{5}{16} = R.H.S.$$

★★ find the value of :—

(i.) $\sin\left(22\frac{1}{2}^\circ\right)$

Ans:

$$\sin^2(\theta) = \frac{1 - \cos(2\theta)}{2}$$

$$\Rightarrow \sin^2\left(22\frac{1}{2}\right) = \frac{1 - \cos(45^\circ)}{2}$$

$$= \frac{\left(1 - \frac{1}{\sqrt{2}}\right)}{2} = \frac{\sqrt{2} - 1}{2\cdot\sqrt{2}}$$

$$\Rightarrow \sin\left(22\frac{1}{2}^\circ\right) = \sqrt{\frac{(\sqrt{2} - 1)}{2\sqrt{2}}}$$

$$\boxed{\therefore \sin\left(22\frac{1}{2}\right)^\circ = \sqrt{\frac{(\sqrt{2} - 1)}{2\cdot\sqrt{2}}}}$$

(ii.) $\cos\left(22\frac{1}{2}\right)^\circ$

Ans:

$$\cos^2(\theta) = \frac{1 + \cos(2\theta)}{2}$$

$$\Rightarrow \cos^2\left(22\frac{1}{2}\right)^\circ = \frac{1 + \cos(45^\circ)}{2}$$

(P.T.O.)

$$= \frac{\left(1+\frac{1}{\sqrt{2}}\right)}{2} = \frac{\sqrt{2}+1}{2\sqrt{2}}$$

$$\Rightarrow \cos\left(22\tfrac{1}{2}\right)^\circ = \sqrt{\frac{\sqrt{2}+1}{2\cdot\sqrt{2}}}$$

$$\boxed{\therefore \cos\left(22\tfrac{1}{2}\right)^\circ = \sqrt{\frac{\sqrt{2}+1}{2\cdot\sqrt{2}}}}$$

(iii) $\tan\left(22\frac{1}{2}\right)^\circ$

Ans:

$$\tan^2\left(22\tfrac{1}{2}^\circ\right) = \frac{\sin^2\left(22\frac{1}{2}^\circ\right)}{\cos^2\left(22\frac{1}{2}^\circ\right)}$$

$$= \frac{\sqrt{2}-1}{2\cdot\sqrt{2}} \times \frac{2\sqrt{2}}{\sqrt{2}+1}$$

$$= \frac{(\sqrt{2}-1)^2}{(\sqrt{2})^2-(1)^2} = \frac{(\sqrt{2}-1)^2}{2-1}$$

$$= (\sqrt{2}-1)^2$$

$$\Rightarrow \tan\left(22\tfrac{1}{2}\right) = (\sqrt{2}-1)$$

$$\boxed{\therefore \tan\left(22\tfrac{1}{2}\right)^\circ = \sqrt{2}-1}$$

Q. Prove that:—

$$\frac{\cos(2x)}{\cos(x)-\sin(x)} = \cos(x)+\sin(x)$$

Ans:

L.H.S.

$$= \frac{\cos(2x)}{\cos(x)-\sin(x)}$$

$$\left\{ \because \cos(2x) = \cos^2(x)-\sin^2(x) \right\}$$

$$= \frac{\cos^2(x)-\sin^2(x)}{\cos(x)-\sin(x)}$$

$$= \frac{(\cos(x)+\sin(x))\cdot\cancel{(\cos(x)-\sin(x))}}{\cancel{(\cos(x)-\sin(x))}}$$

$$= \cos(x)+\sin(x) = \text{R.H.S.}$$

Q. Prove that :—

$$\frac{\sin(2x)}{1+\cos(2x)} = \tan(x)$$

Ans:

L.H.S.

$$= \frac{\sin(2x)}{1+\cos(2x)}$$

Ans:- $= \frac{\cancel{2}\cdot\sin(x)\cdot\cancel{\cos(x)}}{\cancel{2}\cdot\cos^{2}(x)}$

$= \frac{\sin(x)}{\cos(x)} = \tan(x) = R.H.S.$

Q. Prove that :-

$$\frac{\sin(2x)}{1-\cos(2x)} = \cot(x)$$

Ans:- L.H.S.

$= \frac{\sin(2x)}{1-\cos(2x)} = \frac{\cancel{2}\cdot\sin(x)\cdot\cos(x)}{\cancel{2}\cdot\sin^{2}(x)}$

$= \frac{\cos(x)}{\sin(x)} = \cot(x) = R.H.S.$

Q. Prove that :-

$$\frac{\tan(2x)}{1+\sec(2x)} = \tan(x)$$

Ans:- L.H.S.

$= \frac{\tan(2x)}{1+\sec(2x)}$

$= \frac{\frac{\sin(2x)}{\cos(2x)}}{1+\frac{1}{\cos(2x)}} = \frac{\frac{\sin(2x)}{\cancel{\cos(2x)}}}{\frac{\cos(2x)+1}{\cancel{\cos(2x)}}}$

$$= \frac{\sin(2x)}{1+\cos(2x)} = \frac{2\cdot\sin(x)\cdot\cancel{\cos(x)}}{2\cdot\cos^2(x)}$$

$$= \frac{\sin(x)}{\cos(x)} = \tan(x) = R.H.S.$$

Q. Prove that :-

$$\sin(2x)\cdot(\tan(x)+\cot(x)) = 2$$

Ans: L.H.S.

$$= \sin(2x)\cdot(\tan(x)+\cot(x))$$

$$= 2\cdot\sin(x)\cdot\cos(x)\left(\frac{\sin(x)}{\cos(x)}+\frac{\cos(x)}{\sin(x)}\right)$$

$$= 2\cdot\cancel{\sin(x)\cdot\cos(x)}\times\left(\frac{\sin^2(x)+\cos^2(x)}{\cancel{\sin(x)\cdot\cos(x)}}\right)$$

$$= 2\times1 = 2 = R.H.S.$$

Q. Prove that :-

$$\operatorname{cosec}(2x)+\cot(2x) = \cot(x)$$

Ans: L.H.S.

$$= \frac{1}{\sin(2x)}+\frac{\cos(2x)}{\sin(2x)}$$

$$= \frac{1+\cos(2x)}{\sin(2x)}$$

$$= \frac{\cancel{2}\cdot\cos^{\cancel{2}}(x)}{\cancel{2}\cdot\sin(x)\cdot\cancel{\cos(x)}} = \frac{\cos(x)}{\sin(x)} = \cot(x)$$

$$= R.H.S.$$

Q Prove that :-

$$\cos(2x) + 2\cdot\sin^2(x) = 1.$$

Ans: L.H.S.

$= \cos(2x) + 2\cdot\sin^2(x)$

$= 1 - 2\cdot\cancel{\sin^2(x)} + \cancel{2\cdot\sin^2(x)}$

$= 1$ $= R.H.S.$

Q. Prove that :-

$$(\sin(x) - \cos(x))^2 = 1 - \sin(2x)$$

Ans: L.H.S.

$= (\sin(x) - \cos(x))^2$

$= \sin^2(x) + \cos^2(x) - 2\cdot\sin(x)\cdot\cos(x)$

$= 1 - \sin(2x)$

$= R.H.S.$

Q. Prove that :-

$$\cot(x) - 2\cdot\cot(2x) = \tan(x)$$

Ans: L.H.S.

$= \cot(x) - 2\cdot\cot(2x)$

$= \dfrac{1}{\tan(x)} - 2\cdot\dfrac{1}{\tan(2x)}$

$= \cancel{\dfrac{\cos(x)}{\sin(x)}} - 2\cdot\cancel{\dfrac{\cos(2x)}{\sin(2x)}}$

$= \dfrac{1}{\tan(x)} - \dfrac{\cancel{2}(1-\tan^2(x))}{\cancel{2}\tan(x)} = \dfrac{\cancel{1}-\cancel{1}+\tan^{\cancel{2}}(x)}{\cancel{\tan(x)}}$

$= \tan(x) = R.H.S.$

Q. Prove that :-

$$\left(\cos^4(x) + \sin^4(x)\right) = \frac{1}{2}\cdot\left(2 - \sin^2(2\cdot x)\right)$$

Ans:

L.H.S.

$$= \left(\cos^2(x)\right)^2 + \left(\sin^2(x)\right)^2$$

$$\left\{\because a^2 + b^2 = (a+b)^2 - 2a\cdot b\right\}$$

$$= \left(\sin^2(x) + \cos^2(x)\right)^2 - 2\cdot\sin^2(x)\cdot\cos^2(x)$$

$$= 1 - 2\cdot\sin^2(x)\cdot\cos^2(x)$$

$$= 1 - \frac{1}{2}\cdot 2\cdot 2\sin^2(x)\cdot\cos^2(x)$$

$$= 1 - \frac{4\sin^2(x)\cos^2(x)}{2}$$

$$= 1 - \frac{\sin^2(2x)}{2}$$

$$= \frac{1}{2}\cdot\left(2 - \sin^2(2x)\right) = \text{R.H.S.}$$

Q. Prove that :-

$$\frac{\cos^3(x) - \sin^3(x)}{\cos(x) - \sin(x)} = \frac{1}{2}\left(2 + \sin(2x)\right)$$

(P.T.O.)

Ans: L.H.S.

$$= \frac{\cos^3(x) - \sin^3(x)}{\cos(x) - \sin(x)}$$

$$= \frac{\cancel{(\cos(x) - \sin(x))} \cdot (\cos^2(x) + \cos(x)\cdot\sin(x) + \sin^2(x))}{\cancel{(\cos(x) - \sin(x))}}$$

$$= \cos^2(x) + \cos(x)\cdot\sin(x) + \sin^2(x)$$
$$= (\cos^2(x) + \sin^2(x)) + \cos(x)\cdot\sin(x)$$
$$= 1 + \cos(x)\cdot\sin(x)$$
$$= \frac{1}{2}(2 + 2\cdot\cos(x)\cdot\sin(x))$$
$$= \frac{1}{2}\left(2 + \sin(2x)\right)$$ proved.

Q. To prove that :-

$$\frac{1 - \cos(2x) + \sin(x)}{\sin(2x) + \cos(x)} = \tan(x)$$

Ans: L.H.S.

$$= \frac{1 - \cos(2x) + \sin(x)}{\sin(2x) + \cos(x)}$$

$$= \frac{(1 - \cos(2x)) + \sin(x)}{\sin(2x) + \cos(x)}$$

$$= \frac{2\cdot\sin^2(x) + \sin(x)}{2\cos(x)\cdot\sin(x) + \cos(x)}$$

$$= \frac{\sin(x)\,\cancel{(2\sin(x)+1)}}{\cos(x)\,\cancel{(2\cdot \cos(x)+1)}}$$

$\sin(x)$

$$= \tan(x) = R.H.S.$$

Q. Prove that :—

$$\cos(x)\cdot\cos(2x)\cdot\cos(4x)\cdot\cos(8x) = \frac{\sin(16x)}{16\cdot\sin(x)}$$

Ans : L.H.S.

$$= \cos(x)\cdot\cos(2x)\cdot\cos(4x)\cdot\cos(8x)$$

$$= \frac{1}{2\sin(x)}\left[2\cdot\sin(x)\cdot\cos(x)\cdot\cos(2x)\cdot\cos(4x)\cdot\cos(8x)\right]$$

$$= \frac{1}{2\cdot\sin(x)}\left[(2\sin(x)\cdot\cos(x))\cdot\cos(2x)\cdot\cos(4x)\cdot\cos(8x)\right]$$

$$= \frac{1}{2\cdot\sin(x)}\left[\sin(2x)\cdot\cos(2x)\cdot\cos(4x)\cdot\cos(8x)\right]$$

$$= \frac{1}{2\times2\cdot\sin(x)}\left[(2\sin(2x)\cdot\cos(2x))\cdot\cos(4x)\cdot\cos(8\right.$$

$$= \frac{1}{4\cdot\sin(x)}\left[\sin(4x)\cdot\cos(4x)\cdot\cos(8x)\right]$$

$$= \frac{1}{2\times4\cdot\sin(x)}\left[2\cdot\sin(4x)\cdot\cos(4x)\cdot\cos(8x)\right]$$

$$= \frac{1}{8 \cdot \sin(x)} \cdot \left[\sin(8x) \cdot \cos(8 \cdot x)\right]$$

$$= \frac{1}{2 \times 8 \sin(x)} \cdot \left[2 \sin(8x) \cdot \cos(8x)\right]$$

$$= \frac{1}{16 \cdot \sin(x)} \cdot \left[\sin(16 \cdot x)\right]$$

$$= \frac{\sin(16x)}{16 \cdot \sin(x)} = R.H.S.$$ proved

Q. Prove that :—

$$2 \cdot \sin\left(22\tfrac{1}{2}\right)^\circ \cdot \cos\left(22\tfrac{1}{2}\right)^\circ = \frac{1}{\sqrt{2}}.$$

Ans: L.H.S.

$$= 2 \cdot \sin\left(22\tfrac{1}{2}\right)^\circ \cdot \cos\left(22\tfrac{1}{2}\right)^\circ$$

$$= \sin \cancel{2}\left(\frac{45}{\cancel{2}}\right) = \sin 45^\circ = \frac{1}{\sqrt{2}} = R.H.S.$$

Q. Prove that :—

$$8 \cdot \cos^3(20^\circ) - 6 \cdot \cos(20^\circ) = 1.$$

Ans:

$$= 2\left(4 \cdot \cos^3(20^\circ) - 3 \cdot \cos(20^\circ)\right)$$

$$= 2\left[\cos 3(20^\circ)\right]$$

$$= 2\left[\cos 60^\circ\right]$$

$$= \cancel{2} \times \frac{1}{\cancel{2}} = 1$$

$$= R.H.S.$$

proved

Date
Page

Q. Prove that :-

$$3\cdot\sin(40^\circ) - 4\cdot\sin^3 40^\circ = \frac{\sqrt{3}}{2}$$

Ans: L.H.S.

$= 3\cdot\sin(40^\circ) - 4\cdot\sin^3(40^\circ)$

$= \sin 3(40^\circ)$

$= \sin(120^\circ)$

$= \sin(180^\circ - 60^\circ)$

$= \sin 60^\circ$

$= \frac{\sqrt{3}}{2} =$ R.H.S. proved

Q. Prove that :-

$$\sin^2(24^\circ) - \sin^2(6^\circ) = \frac{(\sqrt{5}-1)}{8}$$

Ans: L.H.S.

$= \sin^2(24^\circ) - \sin^2(6^\circ)$

$= \sin(24^\circ+6^\circ)\cdot\sin(24^\circ-6^\circ)$

$= \sin(30^\circ)\cdot\sin(18^\circ)$

$= \frac{1}{2} \times \frac{\sqrt{5}-1}{4} = \frac{\sqrt{5}-1}{8}$

= R.H.S.

Q. Prove that :-

$$\tan 6^\circ\cdot\tan 42^\circ\cdot\tan 66^\circ\cdot\tan 78^\circ = 1$$

(P.T.O.)

Ans: L.H.S.

$= \tan(6^\circ)\cdot\tan(42^\circ)\cdot\tan(66^\circ)\cdot\tan(78^\circ)$

$= \dfrac{\tan(6^\circ)\cdot\tan(42^\circ)\cdot\tan(66^\circ)\cdot\tan(78^\circ)}{\tan(54^\circ)\cdot\tan(18^\circ)} \times \tan(54^\circ)\cdot\tan(18^\circ)$

$= \dfrac{(\tan(6^\circ)\cdot\tan(54^\circ)\cdot\tan(66^\circ))(\tan(18^\circ)\cdot\tan(42^\circ)\cdot\tan(72^\circ))}{\tan(54^\circ)\cdot\tan(18^\circ)}$

$= \dfrac{[\tan 3(6^\circ)]\cdot[\tan 3(18^\circ)]}{\tan(54^\circ)\cdot\tan(18^\circ)}$

$= \dfrac{\cancel{\tan(18^\circ)}\cdot\tan(54^\circ)}{\tan(54^\circ)\cdot\cancel{\tan(18^\circ)}} = 1 = R.H.S.$

proved

Q. If $\tan(x) = -\frac{4}{3}$ and $\frac{\pi}{2} < x < \pi$, find the values of :-

(i) $\sin\left(\frac{x}{2}\right)$ (ii) $\cos\left(\frac{x}{2}\right)$ (iii) $\tan\left(\frac{x}{2}\right)$

Ans: $\because$ x lies in quadrant IInd, we have $\cos(x) < 0$

$\because \tan(x) = -\frac{4}{3}$

$\Rightarrow \sec^2(x) = (1 + \tan^2(x))$

$= 1 + \frac{16}{9}$

$= \frac{25}{9}$

$$\Rightarrow \cos^2(x) = \frac{1}{\sec^2(x)} = \frac{9}{25}$$

$$\Rightarrow \cos(x) = -\sqrt{\frac{9}{25}} = -\frac{3}{5}$$

क्योंकि $\cos(x)$ IInd quadrant में है और हमें यह पहले से पता है की IInd quadrant में cos -ve है।

also, $\frac{\pi}{2} < x < \pi$

$$\Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$$

this implies that $\frac{x}{2}$ lies in quadrant 1.

$$\Rightarrow \sin\left(\frac{x}{2}\right) > 0 \text{ and } \cos\left(\frac{x}{2}\right) > 0.$$

(i) $2\sin^2\left(\frac{x}{2}\right) = (1 - \cos(x))$

$$2\sin^2\left(\frac{x}{2}\right) = 1 - \left(-\frac{3}{5}\right)$$

$$2\sin^2\left(\frac{x}{2}\right) = 1 + \frac{3}{5} = \frac{8}{5}$$

$$\Rightarrow \sin^2\left(\frac{x}{2}\right) = \frac{8}{10}$$

$$\Rightarrow \sin\left(\frac{x}{2}\right) = +\sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$$

{क्योंकी $\frac{\pi}{2} < x < \pi \Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$} इसलिए positive, क्योंकि sine is +ve in Ist quad

(ii.) $\cos\left(\frac{x}{2}\right)$

$$2\cdot\cos^2\left(\frac{x}{2}\right) = \left(1+\cos(x)\right)$$

$$= \left(1-\frac{3}{5}\right) = \frac{2}{5}.$$

$$\cos^2\left(\frac{x}{2}\right) = \frac{2}{5\times 2} = \frac{1}{5}.$$

$$\cos\left(\frac{x}{2}\right) = +\sqrt{\frac{1}{5}} = \frac{1}{\sqrt{5}} \quad \left\{\because \cos\left(\frac{x}{2}\right) > 0\right\}$$

(iii.) $\tan\left(\frac{x}{2}\right)$

$$\tan\left(\frac{x}{2}\right) = \frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)} = \left(\frac{2}{\cancel{\sqrt{5}}} \cancel{\times \sqrt{5}}\right) = 2.$$

Q. If $\cos(x) = -\frac{1}{3}$ and x lies in Quadrant III, find the values of :—

(i) $\sin\left(\frac{x}{2}\right)$ (ii.) $\cos\left(\frac{x}{2}\right)$ (iii) $\tan\left(\frac{x}{2}\right)$

Ans: $\because$ x lies in Quadrant III, we have

$$\pi < x < \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$$

$\Rightarrow \left(\frac{x}{2}\right)$ lies in Quadrant IInd

$$\Rightarrow \sin\left(\frac{x}{2}\right) > 0 \text{ and } \cos\left(\frac{x}{2}\right) < 0$$

(i) $2\cdot\sin^2\left(\frac{x}{2}\right)$

$= 1-\cos(x)$

$= \left(1+\frac{1}{3}\right) = \frac{4}{3}$

$\Rightarrow 2\cdot\sin^2\left(\frac{x}{2}\right) = \frac{4}{3}$

$\Rightarrow \sin^2\left(\frac{x}{2}\right) = \frac{\cancel{4}\,2}{\cancel{2}\times 3}$

$\Rightarrow \sin\left(\frac{x}{2}\right) = +\sqrt{\frac{2}{3}}$

$\Rightarrow \sin\left(\frac{x}{2}\right) = \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$

क्योंकि $\sin\left(\frac{x}{2}\right)$ IInd quadrant में है, और हमें अच्छे से पता है की IInd quadrant में $\sin(x)$ +ve होता है।

(ii) $2\cdot\cos^2\left(\frac{x}{2}\right) = \left(1+\cos(x)\right)$

$= \left(1-\frac{1}{3}\right) = \frac{2}{3}$

$\Rightarrow \cos^2\left(\frac{x}{2}\right) = \frac{1}{3}$

$\Rightarrow \cos\left(\frac{x}{2}\right) = -\frac{1}{\sqrt{3}}$

क्योंकि $\cos\left(\frac{x}{2}\right)$ IInd quadrant में है, और हमें पता है की IInd quadrant में $\cos(x)$ -ve होता है।

(iii) $\tan\left(\frac{x}{2}\right)$

$$= \frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)} = \frac{\sqrt{6}}{3} \times \frac{3}{-\sqrt{3}} = -\sqrt{2}.$$

Q. Prove that :-

$$\frac{1+\cos(x)}{1-\cos(x)} = \left(\operatorname{cosec}(x) + \cot(x)\right)^2$$

Ans: L.H.S.

$$= \frac{1+\cos(x)}{1-\cos(x)} = \frac{\cancel{2}\cdot\cos^2\left(\frac{x}{2}\right)}{\cancel{2}\cdot\sin^2\left(\frac{x}{2}\right)} = \cot^2\left(\frac{x}{2}\right)$$

R.H.S.

$$= \left(\operatorname{cosec}(x) + \cot(x)\right)^2$$

$$= \left(\frac{1}{\sin(x)} + \frac{\cos(x)}{\sin(x)}\right)^2$$

$$= \left(\frac{1+\cos(x)}{\sin(x)}\right)^2$$

$$= \left\{\frac{\cancel{2}\cdot\cos^{\cancel{2}}\left(\frac{x}{2}\right)}{\cancel{2}\cdot\sin\left(\frac{x}{2}\right)\cdot\cancel{\cos\left(\frac{x}{2}\right)}}\right\}^2$$

$$= \left\{\frac{\cos\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)}\right\}^2 = \cot^2\left(\frac{x}{2}\right)$$

$\therefore$ L.H.S. = R.H.S. Proved

Q. Prove that :-

$$\frac{\cos(x)}{(1-\sin(x))} = \tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$$

Ans: L.H.S.

$$= \frac{\left(\cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right)\right)}{\left(\cos^2\left(\frac{x}{2}\right) + \sin^2\left(\frac{x}{2}\right) - 2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)\right)}$$

$$= \frac{\cancel{\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right)} \cdot \left(\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)\right)}{\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right)^{\cancel{2}}}$$

$$= \frac{\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)}$$

$$= \frac{1+\tan\left(\frac{x}{2}\right)}{1-\tan\left(\frac{x}{2}\right)} \quad \left\{\begin{array}{c}\text{dividing num. } \\ \text{\& den. by} \\ \cos\left(\frac{x}{2}\right)\end{array}\right\}$$

$$= \tan\left(\frac{\pi}{4}+\frac{x}{2}\right) = R.H.S.$$

Q. Prove that :-

$$\cot\left(\frac{x}{2}\right) - \tan\left(\frac{x}{2}\right) = 2\cdot\cot(x).$$

Ans: L.H.S.

$$= \cot\left(\frac{x}{2}\right) - \tan\left(\frac{x}{2}\right)$$

$$= \frac{\cos\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)} - \frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}$$

$$= \frac{\cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)\cdot\cos\left(\frac{x}{2}\right)}$$

$$= \frac{2\cos(x)}{2\cdot\sin\left(\frac{x}{2}\right)\cdot\cos\left(\frac{x}{2}\right)}$$

$$= 2.\frac{\cos(x)}{\sin(x)}$$

$$= 2.\cot(x)$$

$$= R.H.S.$$

proved

Q: Prove that :-

$$\tan\left(\frac{\pi}{4}+\frac{x}{2}\right) = \tan(x) + \sec(x)$$

Ans:

L.H.S.

$$= \tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$$

$$= \frac{\tan\left(\frac{\pi}{4}\right) + \tan\left(\frac{x}{2}\right)}{1 - \tan\left(\frac{\pi}{4}\right)\cdot\tan\left(\frac{x}{2}\right)}$$

$$= \frac{1 + \tan(x/2)}{1 - \tan(x/2)}$$

$$= \frac{1 + \frac{\sin(x/2)}{\cos(x/2)}}{1 - \frac{\sin(x/2)}{\cos(x/2)}}$$

$$= \frac{\cos(x/2) + \sin(x/2)}{\cos(x/2) - \sin(x/2)}$$

$$= \frac{\cos(x/2) + \sin(x/2)}{\cos(x/2) - \sin(x/2)} \times \frac{\cos(x/2) + \sin(x/2)}{\cos(x/2) + \sin(x/2)}$$

$$= \frac{\left(\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)\right)^2}{\cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right)}$$

$$= \frac{\cos^2\left(\frac{x}{2}\right) + \sin^2\left(\frac{x}{2}\right) + 2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}{\cos(x)}$$

$$= \frac{1 + \sin(x)}{\cos(x)}$$

$$= \frac{1}{\cos(x)} + \frac{\sin(x)}{\cos(x)}$$

$$= \sec(x) + \tan(x)$$

$= R.H.S$ proved

Q. Prove that :-

$$\sqrt{\frac{1+\sin(x)}{1-\sin(x)}} = \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$$

Ans: L.H.S.

$$= \sqrt{\frac{1+\sin(x)}{1-\sin(x)} \times \frac{1+\sin(x)}{1+\sin(x)}}$$

$$= \sqrt{\frac{(1+\sin(x))^2}{1-\sin^2(x)}}$$

$$= \frac{1+\sin(x)}{\cos^2(x)} = \frac{1+2\cdot\sin\left(\frac{x}{2}\right)\cdot\cos\left(\frac{x}{2}\right)}{\cos(x)}$$

$$= \frac{\cos^2\left(\frac{x}{2}\right) + \sin^2\left(\frac{x}{2}\right) + 2 \cdot \cos\left(\frac{x}{2}\right) \sin\left(\frac{x}{2}\right)}{\cos(x)}$$

$$= \frac{\left(\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)\right)^2}{\cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right)}$$

$$= \frac{\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)}$$

$$= \frac{1 + \frac{\sin(x/2)}{\cos(x/2)}}{1 - \frac{\sin(x/2)}{\cos(x/2)}}$$

$$= \frac{\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)}$$

$$= \frac{1 + \tan\left(\frac{x}{2}\right)}{1 - \tan\left(\frac{x}{2}\right)}$$

$$= \frac{\tan(\pi/4) + \tan(x/2)}{1 - \tan(\pi/4) \cdot \tan(x/2)}$$

$$= \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) = \text{R.H.S. proved}$$

Q. Prove that :-

$$\tan\left(\frac{\pi}{4} + \frac{x}{2}\right) + \tan\left(\frac{\pi}{4} - \frac{x}{2}\right) = 2 \cdot \sec(x)$$

Ans:

L.H.S.

$$= \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) + \tan\left(\frac{\pi}{4} - \frac{x}{2}\right)$$

$$= \frac{\tan(\pi/4) + \tan(x/2)}{1 - \tan(\pi/4) \cdot \tan(x/2)} + \frac{\tan(\pi/4) - \tan(x/2)}{1 + \tan(\pi/4) \cdot \tan(x/2)}$$

$$= \frac{1 + \tan(x/2)}{1 - \tan(x/2)} + \frac{1 - \tan(x/2)}{1 + \tan(x/2)}$$

$$= \frac{1 + \frac{\sin(x/2)}{\cos(x/2)}}{1 - \frac{\sin(x/2)}{\cos(x/2)}} + \frac{1 - \frac{\sin(x/2)}{\cos(x/2)}}{1 + \frac{\sin(x/2)}{\cos(x/2)}}$$

$$= \frac{\frac{\sin(x/2) + \cos(x/2)}{\cos(x/2)}}{\frac{\cos(x/2) - \sin(x/2)}{\cos(x/2)}} + \frac{\frac{\cos(x/2) - \sin(x/2)}{\cos(x/2)}}{\frac{\cos(x/2) + \sin(x/2)}{\cos(x/2)}}$$

$$= \frac{\left(\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)\right)^2 + \left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right)^2}{\cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right)}$$

$$= \frac{2}{\cos(x)} = 2 \cdot \sec(x)$$

$= R.H.S.$ proved

Q. Prove that :-

$$\frac{\sin(x)}{1+\cos(x)} = \tan\left(\frac{x}{2}\right)$$

Ans:

L.H.S.

$$= \frac{\sin(x)}{1+\cos(x)}$$

$$= \frac{\cancel{2} \cdot \sin\left(\frac{x}{2}\right) \cdot \cancel{\cos\left(\frac{x}{2}\right)}}{\cancel{2} \cdot \cos^{\cancel{2}}\left(\frac{x}{2}\right)}$$

$$= \frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}$$

$$= \tan\left(\frac{x}{2}\right) = R.H.S.$$

proved

Q. If $A+B+C=\pi$, prove that

$$\sin(2A)+\sin(2B)+\sin(2C)=4\cdot\sin(A)\cdot\sin(B)\cdot\sin(C)$$

Ans: L.H.S.

$$=\left(\sin(2A)+\sin(2B)\right)+\sin(2C)$$

$$=2\cdot\sin(A+B)\cdot\cos(A-B)+2\cdot\sin C\cdot\cos C$$

$$=2\cdot\sin(\pi-C)\cdot\cos(A-B)+2\cdot\sin C\cdot\cos C$$

$$\left\{\begin{aligned} &\because A+B+C=\pi \\ &(A+B)=\pi-C \end{aligned}\right\}$$

$$=2\cdot\sin(C)\cdot\cos(A-B)+2\cdot\sin C\cdot\cos C$$

$$=2\cdot\sin(C)\left[\cos(A-B)+\cos(C)\right]$$

$$=2\cdot\sin(C)\left[\cos(A-B)+\cos(\pi-(A+B))\right]$$

$$\left\{\begin{aligned} &\because A+B+C=\pi \\ &C=\pi-(A+B) \end{aligned}\right\}$$

$$=2\cdot\sin(C)\left[\cos(A-B)-\cos(A+B)\right]$$

$$=2\cdot\sin(C)\left[2\cdot\sin(A)\cdot\sin(B)\right]$$

$$=4\cdot\sin(A)\cdot\sin(B)\cdot\sin(C)$$

$$=R.H.S.$$

proved

Q. If $A+B+C=\pi$, prove that

$$\sin(2A)-\sin(2B)+\sin(2C)=4\cdot\cos(A)\cdot\sin(B)\cdot\cos(C).$$

Ans:

L.H.S.

$$=(\sin(2A)-\sin(2B))+\sin(2C)$$

$$=2\cdot\cos(A+B)\cdot\sin(A-B)+2\sin C\cdot\cos C$$

$$\left\{\begin{array}{l}\because A+B+C=\pi \\ A+B=(\pi-C)\end{array}\right\}$$

$$=2\cdot\cos(\pi-C)\cdot\sin(A-B)+2\sin C\cdot\cos C$$

$$=-2\cdot\cos C\cdot\sin(A-B)+2\sin C\cdot\cos C$$

$$=-2\cos C\left[\sin(A-B)-\sin(C)\right]$$

$$=-2\cos C\left[\sin(A-B)-\sin\{\pi-(A+B)\}\right]$$

$$\left\{\begin{array}{l}\because A+B+C=\pi \\ C=\pi-(A+B)\end{array}\right\}$$

$$=-2\cdot\cos C\left[\sin(A-B)-\sin(A+B)\right]$$

$$=-2\cdot\cos C\left[-2\cdot\cos A\cdot\sin B\right]$$

$$=4\cdot\cos A\cdot\sin B\cdot\cos C$$

$$=\text{R.H.S.}$$

Proved

Q. If $A+B+C=\pi$, prove that

$\cos(2\cdot A)+\cos(2\cdot B)+\cos(2\cdot C)=-1-4\cdot\cos A\cdot\cos B\cdot\cos C$

Ans: L.H.S.

$= \cos(2A)+\cos(2\cdot B)+\cos(2\cdot C)$

$= 2\cdot\cos(A+B)\cdot\cos(A-B)+2\cdot\cos^2(C)-1$

$= 2\cdot\cos(\pi-C)\cdot\cos(A-B)+2\cos^2(C)-1$

$= -2\cdot\cos C\cdot\cos(A-B)+2\cdot\cos^2(C)-1$

$= -2\cdot\cos C\,[\cos(A-B)-\cos(C)]-1$

$= -1-2\cdot\cos C\,[\cos(A-B)-\cos\{\pi-(A+B)\}]$

$= -1-2\cdot\cos C\,[\cos(A-B)+\cos(A+B)]$

$= -1-2\cdot\cos C\,[2\cdot\cos A\cdot\cos B]$

$= -1-4\cdot\cos A\cdot\cos B\cdot\cos C$ = R.H.S.

Q. If $A+B+C=\pi$, prove that

$\cos(2\cdot A)+\cos(2B)-\cos(2C)=$

$1-4\cdot\sin(A)\cdot\sin(B)\cdot\cos(C)$

Ans: L.H.S.

$= (\cos(2A)+\cos(2\cdot B))-\cos(2C)$

$= 2\cdot\cos(A+B)\cdot\cos(A-B)-\cos(2C)$

$= 2\cdot\cos(\pi-C)\cdot\cos(A-B)-2\cos^2(C)-1$

$\left\{\because A+B+C=\pi \atop A+B=\pi-C\right\}$

White Page

Date
Page

$= -2\cdot\cos(C)\cdot\cos(A-B) - 2\cdot\cos^2(C) + 1$

$= 1 - 2\cos(C)\,[\cos(A-B) + \cos(C)]$

$= 1 - 2\cdot\cos(C)\,[\cos(A-B) + \cos\{\pi-(A+B)\}]$

$$\left\{\begin{array}{l} \because A+B+C = \pi \\ \quad C = \pi - (A+B) \end{array}\right\}$$

$= 1 - 2\cdot\cos(C)\,[\cos(A-B) - \cos(A+B)]$

$= 1 - 2\cdot\cos(C)\,[2\cdot\sin(A)\cdot\sin(B)]$

$= 1 - 4\cdot\sin A\cdot\sin B\cdot\cos C$

$= R\cdot H\cdot S.$

Q. If $A+B+C=\pi$, prove that

$\cos(4A) + \cos(4B) + \cos(4C) =$
$-1 + 4\cdot\cos(2A)\cdot\cos(2B)\cdot\cos(2C)$

Ans: $L\cdot H\cdot S.$

$= \cos(4A) + \cos(4B) + \cos(4\cdot C)$

$= 2\cdot\cos(2A+2B)\cdot\cos(2A-2B) + \cos(4\cdot C)$

$= 2\cdot\cos(2\pi - 2C)\cdot\cos(2A-2B) + (2\cdot\cos^2(2C) - 1)$

$$\left\{\begin{array}{l} \because A+B+C=\pi \\ A+B = \pi - C \\ 2A+2B = 2\pi - 2C \end{array}\right\}$$

$= 2\cdot\cos(2C)\cdot\cos(2A-2B) + 2\cos^2(2C) - 1$

$= 2\cdot\cos(2C)\,[\cos(2A-2B) + \cos(2C)] - 1$

$$= 2\cdot\cos(2C)\cdot\left[\cos(2A-2B) + \cos\{2\pi - (2A+2B)\}\right] - 1$$

$$\left\{\begin{aligned} \because\ A+B+C &= \pi \\ 2A+2B+2C &= 2\pi \\ 2C &= 2\pi - (2A+2B) \end{aligned}\right\}$$

$$= 2\cdot\cos(2C)\left[\cos(2A-2B) + \cos(2A+2B)\right] - 1$$

$$= 2\cdot\cos(2C)\cdot\left[2\cdot\cos(2A)\cdot\cos(2B)\right] - 1$$

$$= 4\cdot\cos(2A)\cdot\cos(2B)\cdot\cos(2C) - 1.$$

$= R.H.S.$ proved

Q. Prove that :—

If $A+B+C = \pi$

$$\sin(A) + \sin(B) - \sin(C) = 4\cdot\sin\left(\frac{A}{2}\right)\cdot\sin\left(\frac{B}{2}\right)\cdot\cos\left(\frac{C}{2}\right)$$

Ans: L.H.S.

$$= (\sin(A) + \sin(B)) - \sin(C)$$

$$= 2\sin\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right) - 2\sin\left(\frac{C}{2}\right)\cdot\cos\left(\frac{C}{2}\right)$$

$$\left\{\begin{aligned} \because\ A+B+C &= \pi \\ \frac{A+B+C}{2} &= \frac{\pi}{2} \\ \frac{A+B}{2} &= \frac{\pi}{2} - \frac{C}{2} \end{aligned}\right\}$$

White Page

Date
Page

$$= 2\cdot\sin\left(\frac{\pi}{2}-\frac{C}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right) - 2\cdot\sin\left(\frac{C}{2}\right)\cdot\cos\left(\frac{C}{2}\right)$$

$$= 2\cdot\cos\left(\frac{C}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right) - 2\cdot\sin\left(\frac{C}{2}\right)\cdot\cos\left(\frac{C}{2}\right)$$

$$= 2\cdot\cos\left(\frac{C}{2}\right)\left[\cos\left(\frac{A-B}{2}\right) - \sin\left(\frac{C}{2}\right)\right]$$

$$\left\{\begin{aligned} &\because A+B+C = \pi \\ &\frac{A+B+C}{2} = \frac{\pi}{2} \\ &\frac{C}{2} = \frac{\pi}{2} - \left(\frac{A+B}{2}\right) \end{aligned}\right\}$$

$$= 2\cdot\cos\left(\frac{C}{2}\right)\left[\cos\left(\frac{A-B}{2}\right) - \sin\left\{\frac{\pi}{2} - \left(\frac{A+B}{2}\right)\right\}\right]$$

$$= 2\cdot\cos\left(\frac{C}{2}\right)\cdot\left[\cos\left(\frac{A-B}{2}\right) - \cos\left(\frac{A+B}{2}\right)\right]$$

$$= 2\cdot\cos\left(\frac{C}{2}\right)\left[2\cdot\sin\left(\frac{A}{2}\right)\cdot\sin\left(\frac{B}{2}\right)\right]$$

$$= 4\cdot\sin\left(\frac{A}{2}\right)\cos\left(\frac{C}{2}\right)\sin\left(\frac{B}{2}\right)$$

$= R.H.S$

proved

Q. If $A+B+C=\pi$, prove that

$$\cos(A)+\cos(B)-\cos(C)=\left(4\cdot\cos\left(\frac{A}{2}\right)\cdot\cos\left(\frac{B}{2}\right)\cdot\sin\left(\frac{C}{2}\right)\right)-1$$

Ans:

L.H.S.

$$=\cos(A)+\cos(B)-\cos(C)$$

$$=2\cdot\cos\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)-\left(1-2\cdot\sin^2\left(\frac{C}{2}\right)\right)$$

$$\left\{\begin{array}{l}\because A+B+C=\pi \\ \Rightarrow \frac{A+B+C}{2}=\frac{\pi}{2} \\ \Rightarrow \frac{A+B}{2}=\frac{\pi}{2}-\frac{C}{2}\end{array}\right\}$$

$$=2\cdot\cos\left(\frac{\pi}{2}-\frac{C}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)-\left[1-2\cdot\sin^2\left(\frac{C}{2}\right)\right]$$

$$=2\cdot\sin\left(\frac{C}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)-1+2\sin^2\left(\frac{C}{2}\right)$$

$$=2\sin\left(\frac{C}{2}\right)\left[\cos\left(\frac{A-B}{2}\right)+\sin\left(\frac{C}{2}\right)\right]-1$$

$$\left\{\begin{array}{l}\because A+B+C=\pi \\ \Rightarrow \frac{A+B+C}{2}=\frac{\pi}{2} \\ \Rightarrow \frac{C}{2}=\frac{\pi}{2}-\left(\frac{A+B}{2}\right)\end{array}\right\}$$

White Page

Date

Page

$$= 2\cdot\sin\left(\frac{C}{2}\right)\cdot\left[\cos\left(\frac{A-B}{2}\right)+\sin\left\{\frac{\pi}{2}-\frac{(A+B)}{2}\right\}\right]-1$$

$$= 2\cdot\sin\left(\frac{C}{2}\right)\cdot\left[\cos\left(\frac{A-B}{2}\right)+\cos\left(\frac{A+B}{2}\right)\right]-1$$

$$= 2\cdot\sin\left(\frac{C}{2}\right)\left\{2\cdot\cos\left(\frac{A}{2}\right)\cdot\cos\left(\frac{B}{2}\right)\right\}-1$$

$$= \left\{4\cdot\cos\left(\frac{A}{2}\right)\cdot\cos\left(\frac{B}{2}\right)\cdot\sin\left(\frac{C}{2}\right)\right\}-1$$

$= R.H.S$

Q. If $A+B+C=\pi$, prove that

$$\sin^2(A)+\sin^2(B)+\sin^2(C) = 2\cdot(1+\cos(A)\cdot\cos(B)\cdot\cos(C))$$

Ans: L.H.S.

$$= \sin^2(A)+\sin^2(B)+\sin^2(C)$$

$$= \frac{1}{2}(1-\cos(2A))+\frac{1}{2}(1-\cos(2B))+\frac{1}{2}(1-\cos(2C))$$

$$= \frac{3}{2}-\frac{1}{2}(\cos(2A)+\cos(2B)+\cos(2C))$$

$$= \frac{3}{2}-\frac{1}{2}(-1-4\cdot\cos A\cdot\cos B\cdot\cos C)$$

$$= 2+2\cdot\cos A\cdot\cos B\cdot\cos C$$

$$= 2(1+\cos A\cdot\cos B\cdot\cos C)$$

$= R.H.S.$

White Page

Date ____
Page ____

Q. If, $A+B+C=\pi$, prove that

$$\cos^2(A)+\cos^2(B)-\cos^2(C)=1-2\cdot\sin(A)\cdot\sin(B)\cdot\cos(C)$$

Ans:

L.H.S.

$$=\cos^2(A)+\cos^2(B)-\cos^2(C)$$

$$=\frac{1}{2}\left(1+\cos(2A)\right)+\frac{1}{2}\left(1+\cos(2B)\right)-\frac{1}{2}\left(1+\cos(2C)\right)$$

$$=\frac{1}{2}+\frac{1}{2}\cdot\left(\cos(2A)+\cos(2B)-\cos(2C)\right)$$

$$=\frac{1}{2}+\frac{1}{2}\left(1-4\sin(A)\cdot\sin(B)\cdot\cos(C)\right)$$

$$=1-2\sin(A)\cdot\sin(B)\cdot\cos(C)$$

$=$ R.H.S. Proved//

Q. In a ΔABC, prove that

$$\cos^2(A)+\cos^2\left(A+\frac{\pi}{3}\right)+\cos^2\left(A-\frac{\pi}{3}\right)=\frac{3}{2}$$

Ans:

L.H.S.

$$=\cos^2(A)+\cos^2\left(A+\frac{\pi}{3}\right)+\cos^2\left(A-\frac{\pi}{3}\right)$$

$$=\frac{1}{2}\left(1+\cos(2A)\right)+\frac{1}{2}\left\{1+\cos\left(2A+\frac{2\pi}{3}\right)\right\}$$

$$+\frac{1}{2}\left\{1+\cos\left(2A-\frac{2\pi}{3}\right)\right\}$$

$$= \frac{3}{2} + \frac{1}{2}\cdot\cos(2A) + \frac{1}{2}\left\{\cos\left(2A + \frac{2\pi}{3}\right) + \cos\left(2A - \frac{2\pi}{3}\right)\right\}$$

$$= \frac{3}{2} + \frac{1}{2}\cdot\cos(2A) + \cos(2A)\cdot\cos\left(\frac{2\pi}{3}\right)$$

$$= \frac{3}{2} + \frac{1}{2}\cdot\cos(2A) + \cos(2A) \times -\frac{1}{2}$$

$$= \frac{3}{2} + \cancel{\frac{1}{2}\cdot\cos(2A)} - \cancel{\frac{1}{2}\cdot\cos(2A)}$$

$$= \frac{3}{2} = \text{R.H.S.} \quad \text{proved}$$

Q. If $A+B+C = \pi$, prove that

$$\sin^2\left(\frac{A}{2}\right) + \sin^2\left(\frac{B}{2}\right) - \sin^2\left(\frac{C}{2}\right) = 1 - 2\cdot\cos\left(\frac{A}{2}\right)\cdot\cos\left(\frac{B}{2}\right)\cdot\sin\left(\frac{C}{2}\right)$$

Ans:

L.H.S.

$$= \sin^2\left(\frac{A}{2}\right) + \sin^2\left(\frac{B}{2}\right) - \sin^2\left(\frac{C}{2}\right)$$

$$= \frac{1}{2}(1-\cos(A)) + \frac{1}{2}(1-\cos(B)) - \frac{1}{2}(1-\cos(C))$$

$$= \frac{1}{2} - \frac{1}{2}(\cos(A) + \cos(B) + \cos(C))$$

$$= \frac{1}{2} - \frac{1}{2}\left[\left(4\cdot\cos\left(\frac{A}{2}\right)\cdot\cos\left(\frac{B}{2}\right)\cdot\sin\left(\frac{C}{2}\right)\right) - 1\right]$$

$$= 1 - 2\cdot\cos\left(\frac{A}{2}\right)\cdot\cos\left(\frac{B}{2}\right)\cdot\sin\left(\frac{C}{2}\right)$$

$$= \text{R.H.S}$$

White Page

Date

Page

Q. If $A+B+C=\pi$, prove that

$$\cos^2\left(\frac{A}{2}\right)+\cos^2\left(\frac{B}{2}\right)-\cos^2\left(\frac{C}{2}\right)=2\cdot\cos\left(\frac{A}{2}\right)\cdot\cos\left(\frac{B}{2}\right)\cdot\sin\left(\frac{C}{2}\right)$$

Ans:

L.H.S.

$$=\cos^2\left(\frac{A}{2}\right)+\cos^2\left(\frac{B}{2}\right)-\cos^2\left(\frac{C}{2}\right)$$

$$=\frac{1}{2}(1+\cos(A))+\frac{1}{2}(1+\cos(B))-\frac{1}{2}(1+\cos(C))$$

$$=\frac{1}{2}+\frac{1}{2}(\cos(A)+\cos(B)-\cos(C))$$

$$=\frac{1}{2}+\frac{1}{2}\left[\left(4\cdot\cos\left(\frac{A}{2}\right)\cdot\cos\left(\frac{B}{2}\right)\cdot\sin\left(\frac{C}{2}\right)\right)-1\right]$$

$$=2\cdot\cos\left(\frac{A}{2}\right)\cdot\cos\left(\frac{B}{2}\right)\cdot\sin\left(\frac{C}{2}\right)$$

$=$ R.H.S.

Q. If $A+B+C=\pi$, prove that

$$\cos^2\left(\frac{A}{2}\right)+\cos^2\left(\frac{B}{2}\right)+\cos^2\left(\frac{C}{2}\right)=2\left(1+\sin\left(\frac{A}{2}\right)\cdot\sin\left(\frac{B}{2}\right)\cdot\sin\left(\frac{C}{2}\right)\right)$$

Ans:

L.H.S.

$$=\cos^2\left(\frac{A}{2}\right)+\cos^2\left(\frac{B}{2}\right)+\cos^2\left(\frac{C}{2}\right)$$

$$=\frac{1}{2}(1+\cos(A))+\frac{1}{2}(1+\cos(B))+\frac{1}{2}(1+\cos(C))$$

$$=\frac{3}{2}+\frac{1}{2}(\cos(A)+\cos(B)+\cos(C))$$

$= \frac{3}{2} + \frac{1}{2}\left[2\cdot\cos\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right) + \cos C\right]$

$= \frac{3}{2} + \cos\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right) + \frac{1}{2}\cos(C)$

$\left\{ \begin{array}{l} \because A+B+C = \pi \\ \Rightarrow \frac{A+B+C}{2} = \frac{\pi}{2} \\ \Rightarrow \frac{A+B}{2} = \frac{\pi}{2} - \frac{C}{2} \end{array} \right\}$

$= \frac{3}{2} + \cos\left(\frac{\pi}{2} - \frac{C}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right) + \frac{1}{2}\left(1 - 2\sin^2\left(\frac{C}{2}\right)\right)$

$= \frac{3}{2} + \sin\left(\frac{C}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right) + \frac{1}{2} - \sin^2\left(\frac{C}{2}\right)$

$= 2 + \sin\left(\frac{C}{2}\right)\left[\cos\left(\frac{A-B}{2}\right) - \sin\left(\frac{C}{2}\right)\right]$

$\left\{ \begin{array}{l} \because A+B+C = \pi \\ \Rightarrow \frac{A+B+C}{2} = \frac{\pi}{2} \\ \Rightarrow \frac{C}{2} = \frac{\pi}{2} - \left(\frac{A+B}{2}\right) \end{array} \right\}$

(P.T.O.)

Date
Page

$$= 2 + \sin\left(\frac{C}{2}\right)\left[\cos\left(\frac{A-B}{2}\right) - \sin\left\{\frac{\pi}{2} - \left(\frac{A+B}{2}\right)\right\}\right]$$

$$= 2 + \sin\left(\frac{C}{2}\right)\left[\cos\left(\frac{A-B}{2}\right) - \cos\left(\frac{A+B}{2}\right)\right]$$

$$= 2 + \sin\left(\frac{C}{2}\right)\left(2\sin\left(\frac{A}{2}\right)\cdot\sin\left(\frac{B}{2}\right)\right)$$

$$= 2 + 2\cdot\sin\left(\frac{A}{2}\right)\cdot\sin\left(\frac{B}{2}\right)\cdot\sin\left(\frac{C}{2}\right)$$

$$= 2\left(1 + \sin\left(\frac{A}{2}\right)\cdot\sin\left(\frac{B}{2}\right)\cdot\sin\left(\frac{C}{2}\right)\right)$$

= R.H.S proved

Q. If $A+B+C = \pi$, prove that

$\tan(A) + \tan(B) + \tan(C) = \tan(A)\cdot\tan(B)\cdot\tan(C)$

Ans:

$$A + B + C = \pi$$

$$\Rightarrow A + B = (\pi - C)$$

$$\Rightarrow \tan(A+B) = \tan(\pi - C)$$

$$\Rightarrow \frac{\tan(A) + \tan(B)}{1 - \tan(A)\cdot\tan(B)} = -\tan(C),$$

$$\Rightarrow \tan(A) + \tan(B) = -\tan(C) + \tan(A)\cdot\tan(B)\cdot\tan(C)$$

$$\Rightarrow \tan(A) + \tan(B) + \tan(C) = \tan(A)\cdot\tan(B)\cdot\tan(C)$$

= R.H.S proved

White Page

Date

Page

Q If $A+B+C = \pi$, prove that

$$\tan\left(\frac{A}{2}\right)\cdot\tan\left(\frac{B}{2}\right) + \tan\left(\frac{B}{2}\right)\cdot\tan\left(\frac{C}{2}\right) + \tan\left(\frac{C}{2}\right)\cdot\tan\left(\frac{A}{2}\right) = 1.$$

Ans: here,

$$A+B+C = \pi$$

$$\Rightarrow \frac{A+B}{2} = \frac{\pi}{2} - \frac{C}{2}$$

$$\Rightarrow \tan\left(\frac{A+B}{2}\right) = \tan\left(\frac{\pi}{2} - \frac{C}{2}\right) = \cot\left(\frac{C}{2}\right)$$

$$\Rightarrow \tan\left(\frac{A+B}{2}\right) = \cot\left(\frac{C}{2}\right)$$

$$\Rightarrow \frac{\tan\left(\frac{A}{2}\right) + \tan\left(\frac{B}{2}\right)}{1 - \tan\left(\frac{A}{2}\right)\cdot\tan\left(\frac{B}{2}\right)} = \frac{1}{\tan\left(\frac{C}{2}\right)}$$

$$\Rightarrow \tan\left(\frac{A}{2}\right)\cdot\tan\left(\frac{C}{2}\right) + \tan\left(\frac{B}{2}\right)\cdot\tan\left(\frac{C}{2}\right) = 1 - \tan\left(\frac{A}{2}\right)\cdot\tan\left(\frac{B}{2}\right)$$

$$\Rightarrow \tan\left(\frac{A}{2}\right)\cdot\tan\left(\frac{B}{2}\right) + \tan\left(\frac{B}{2}\right)\cdot\tan\left(\frac{C}{2}\right) + \tan\left(\frac{C}{2}\right)\cdot\tan\left(\frac{A}{2}\right) = 1.$$

proved

Q. If $A+B+C = \pi$, prove that

$$\cot(B)\cdot\cot(C) + \cot(C)\cdot\cot(A) + \cot(A)\cdot\cot(B) = 1$$

Ans: here,

$$A+B+C = \pi$$

$$\Rightarrow (A+B) = (\pi - C)$$

$$\Rightarrow \cot(A+B) = \cot(\pi - C)$$

$$\Rightarrow \cot(A+B) = -\cot(C)$$

$$\Rightarrow \frac{\cot(A)\cdot\cot(B) - 1}{\cot(A) + \cot(B)} = -\cot(C)$$

$$\Rightarrow \cot(A)\cdot\cot(B) - 1 = -\cot(A)\cdot\cot(C) - \cot(C)\cdot\cot(B)$$

$$\Rightarrow \cot(A)\cdot\cot(B) + \cot(B)\cdot\cot(C) + \cot(C)\cdot\cot(A) = 1.$$

proved.

Q. If $A+B+C=\pi$, prove that

$$\cot\left(\frac{A}{2}\right) + \cot\left(\frac{B}{2}\right) + \cot\left(\frac{C}{2}\right) = \cot\left(\frac{A}{2}\right)\cdot\cot\left(\frac{B}{2}\right)\cdot\cot\left(\frac{C}{2}\right)$$

Ans: here, $A+B+C = \pi$

$$\Rightarrow \frac{A+B+C}{2} = \frac{\pi}{2}$$

$$\Rightarrow \left(\frac{A}{2} + \frac{B}{2}\right) = \frac{\pi}{2} - \left(\frac{C}{2}\right)$$

$$\Rightarrow \cot\left(\frac{A}{2} + \frac{B}{2}\right) = \cot\left(\frac{\pi}{2} - \left(\frac{C}{2}\right)\right) = \tan\left(\frac{C}{2}\right)$$

$$\Rightarrow \frac{\cot\left(\frac{A}{2}\right)\cdot\cot\left(\frac{B}{2}\right) - 1}{\cot\left(\frac{A}{2}\right) + \cot\left(\frac{B}{2}\right)} = \frac{1}{\cot\left(\frac{C}{2}\right)}$$

$$\Rightarrow \cot\left(\frac{A}{2}\right) + \cot\left(\frac{B}{2}\right) = \cot\left(\frac{A}{2}\right)\cdot\cot\left(\frac{B}{2}\right)\cdot\cot\left(\frac{C}{2}\right) - \cot\left(\frac{C}{2}\right)$$

$$\Rightarrow \cot\left(\frac{A}{2}\right) + \cot\left(\frac{B}{2}\right) + \cot\left(\frac{C}{2}\right) = \cot\left(\frac{A}{2}\right)\cdot\cot\left(\frac{B}{2}\right)\cdot\cot\left(\frac{C}{2}\right)$$

proved

Q. If $A+B+C = \pi$, prove that

$$\sin(2A) + \sin(2B) - \sin(2C) = 4\cdot\cos(A)\cdot\cos(B)\cdot\cancel{\cos(C)}\sin(C)$$

Ans:

L.H.S.

$$= \sin(2A) + \sin(2B) - \sin(2C)$$

$$= 2\cdot\sin(A)\cdot\cos(A) + 2\cdot\sin(B)\cdot\cos(C) - 2\cdot\sin(C)\cdot\cos(A)$$

$\because A+B+C = \pi$

$\cancel{B+C = (\pi - A)}$

$\Rightarrow A = \pi - (B+C)$

$\Rightarrow B = \pi - (A+B)$

$\Rightarrow C = \pi - (A+B)$

$= 2\cdot\sin(\pi-(B+C))\cdot\cos(A) +$
$2\cdot\sin(\pi-(A+C))\cdot\cos(C) -$
$2\cdot\sin(\pi-(A+B))\cdot\cos(A))$

$= 2\cdot\sin(B+C)\cdot\cos(A) + 2\cdot\sin(A+C)\cdot\cos(C) -$
$2\cdot\sin(A+B)\cdot\cos(C).$

$= 2(\sin(B)\cdot\cos(C)+\cos(B)\sin(C))\cdot\cos(A) +$
$2(\sin(A)\cdot\cos(C)+\cos(A)\sin(C))\cdot\cos(B) -$
$2(\sin(A)\cdot\cos(B)+\cos(A)\sin(B))\cos(C)$

$= 2\cdot\cos(A)\cdot\cancel{\sin(B)\cdot\cos(C)} + 2\cos(A)\cdot\cos(B)\cdot\sin(C)$
$+ \cancel{2\cdot\sin(A)\cdot\cos(C)\cdot\cos(B)} + 2\cdot\cos(A)\cdot\sin(C)\cdot\cos(B) -$
$\cancel{2\cdot\sin(A)\cdot\cos(B)\cdot\cos(C)} - 2\cdot\cancel{\cos(A)}\cdot\sin(B)\cdot\cos(C)$

$= 2\cdot\cos(A)\cdot\cos(B)\cdot\sin(C) +$
$2\cdot\cos(A)\cdot\sin(C)\cdot\cos(B)$

$= 2\cos(A)(\cos(B)\cdot\sin(C) + \sin(C)\cdot\cos(B))$

$= 2\cos(A)\cdot(2\cos(B)\cdot\sin(C))$

$= 4\cdot\cos(A)\cdot\cos(B)\cdot\sin(C)$

= R.H.S. proved

Q. If $A+B+C=\pi$, prove that

$\cos(2A)-\cos(2B)-\cos(2C) = -1+4\cdot\cos(A)\cdot\sin(B)\cdot\sin(C)$

Ans: $\because A+B+C=\pi$

L.H.S.

$$= \cos(2A) - (\cos(2B) + \cos(2C))$$

$$= \cos(2A) - \left\{2\cdot\cos\left(\frac{2B+2C}{2}\right)\cdot\cos\left(\frac{2B-2C}{2}\right)\right\}$$

$$= \cos(2A) - \left\{2\cdot\cos(B+C)\cdot\cos(B-C)\right\}$$

$$\left\{\begin{array}{l}\because A+B+C=\pi \\ \Rightarrow (B+C) = (\pi - A)\end{array}\right\}$$

$$= \cos(2A) - \left\{2\cdot\cos(\pi-A)\cdot\cos(B-C)\right\}$$

$$= \cos(2A) - \left\{-2\cdot\cos(A)\cdot\cos(B-C)\right\}$$

$$= \cos(2A) + 2\cdot\cos(A)\cdot\cos(B-C)$$

$$= 2\cdot\cos^2(A) - 1 + 2\cdot\cos(A)\cdot\cos(B-C)$$

$$= 2\cdot\cos(A)\left\{\cos(A) + \cos(B-C)\right\} - 1$$

$$= 2\cdot\cos(A)\left\{2\cdot\cos\left(\frac{A+B-C}{2}\right)\cdot\cos\left(\frac{A+C-B}{2}\right)\right\} - 1$$

$$= 2\cdot\cos(A)\left\{2\cdot\cos\left(\frac{\pi-C-C}{2}\right)\cdot\cos\left(\frac{\pi-B-B}{2}\right)\right\}$$

White Page

Date

Page

$$= 2\cdot\cos(A)\left\{2\cdot\cos\left(\frac{\pi}{2}-\frac{2C}{2}\right)\cdot\cos\left(\frac{\pi}{2}-\frac{2B}{2}\right)\right\}-1$$

$$= 2\cdot\cos(A)\{2\cdot\sin(C)\cdot\sin(B)\}-1$$

$$= 4\cdot\cos(A)\cdot\sin(B)\cdot\sin(C)-1$$

$= R.H.S.$

Q If $A+B+C=\pi$, prove that :-

$$\cos(2A)-\cos(2B)+\cos(2C)=1-4\cdot\sin(A)\cdot\cos(B)\cdot\sin(C)$$

Ans: L.H.S.

$$= \cos(2A)-\cos(2B)+\cos(2C)$$

$$= \cos(2A)-\left\{2\cdot\sin\left(\frac{2B+2C}{2}\right)\cdot\sin\left(\frac{2B-2C}{2}\right)\right\}$$

$$= \cos(2A)-\{2\cdot\sin(B+C)\cdot\sin(B-C)\}$$

$$\left\{\begin{aligned} \because A+B+C &= \pi \\ B+C &= (180^\circ - A) \end{aligned}\right\}$$

$$= \cos(2A)-\{2\cdot\sin(\pi-A)\cdot\sin(B-C)\}$$

$$= \cos(2A)-\{2\cdot\sin(A)\cdot\sin(B-C)\}$$

$$= 1-2\cdot\sin^2 A - 2\sin(A)\cdot\sin(B-C)$$

$$= -2\cdot\sin(A)\{\sin A+\sin(B-C)\}+1$$

White Page

Date

Page

$$= -2\cdot\sin(A)\left\{2\cdot\sin\left(\frac{A+B-C}{2}\right)\cdot\cos\left(\frac{A+C-B}{2}\right)\right\}+1$$

$$\cancel{= -2\cdot\sin(A)}\left\{\because A+B+C=\pi\right.$$

$$\Rightarrow A+B=(\pi-C)$$

$$\left.\Rightarrow A+C=(\pi-B)\right\}$$

$$= -2\cdot\sin(A)\left\{2\cdot\sin\left(\frac{\pi-C-C}{2}\right)\cdot\cos\left(\frac{\pi-B-B}{2}\right)\right\}+1$$

$$= -2\cdot\sin(A)\left\{2\cdot\sin\left(\frac{\pi}{2}-\frac{2C}{2}\right)\cdot\cos\left(\frac{\pi}{2}-\frac{2B}{2}\right)\right\}+1$$

$$= -2\cdot\sin(A)\left\{2\cdot\sin\left(\frac{\pi}{2}-C\right)\cdot\cos\left(\frac{\pi}{2}-B\right)\right\}+1$$

$$= -2\cdot\sin(A)\left\{2\cdot\cos(C)\cdot\sin(B)\right\}+1$$

$$= -4\cdot\sin(A)\cdot\sin(B)\cdot\cos(C)+1$$

$= R.H.S.$

proved

Q. If $A+B+C=\pi$, prove that

$$\sin(A)+\sin(B)+\sin(C)=4\cdot\cos\left(\frac{A}{2}\right)\cdot\cos\left(\frac{B}{2}\right)\cdot\cos\left(\frac{C}{2}\right)$$

Ans: L.H.S.

$$=\sin(A)+\sin(B)+\sin(C)$$

$$=\sin(A)+\left\{2\cdot\sin\left(\frac{B+C}{2}\right)\cdot\cos\left(\frac{B-C}{2}\right)\right\}$$

$$\#\left\{\begin{aligned}&\because A+B+C=\pi\\&\Rightarrow\frac{A+B+C}{2}=\frac{\pi}{2}\\&\Rightarrow\frac{B+C}{2}=\left(\frac{\pi}{2}-\frac{A}{2}\right)\end{aligned}\right\}$$

$$=\sin(A)+\left\{2\cdot\sin\left(\frac{\pi}{2}-\frac{A}{2}\right)\cdot\cos\left(\frac{B-C}{2}\right)\right\}$$

$$=2\cdot\sin\left(\frac{A}{2}\right)\cdot\cos\left(\frac{A}{2}\right)+\left\{2\cdot\cos\left(\frac{A}{2}\right)\cdot\cos\left(\frac{B-C}{2}\right)\right\}$$

$$=2\cdot\cos\left(\frac{A}{2}\right)\left\{\sin\left(\frac{A}{2}\right)+\cos\left(\frac{B-C}{2}\right)\right\}$$

$$\left\{\begin{aligned}&\because A+B+C=\pi\\&\Rightarrow\frac{A+B+C}{2}=\frac{\pi}{2}\\&\Rightarrow\frac{A}{2}=\left\{\frac{\pi}{2}-\left(\frac{B+C}{2}\right)\right\}\end{aligned}\right\}$$

$$= 2\cdot\cos\left(\frac{A}{2}\right)\cdot\left\{\sin\left\{\frac{\pi}{2}-\left(\frac{B+C}{2}\right)\right\}+\cos\left(\frac{B-C}{2}\right)\right\}$$

$$= 2\cdot\cos\left(\frac{A}{2}\right)\cdot\left\{\cos\left(\frac{B+C}{2}\right)+\cos\left(\frac{B-C}{2}\right)\right\}$$

$$= 2\cdot\cos\left(\frac{A}{2}\right)\left\{2\cdot\cos\left(\frac{B}{2}\right)\cdot\cos\left(\frac{C}{2}\right)\right\}$$

$$= 4\cdot\cos\left(\frac{A}{2}\right)\cdot\cos\left(\frac{B}{2}\right)\cdot\cos\left(\frac{C}{2}\right)$$

= R.H.S. proved

Q. If $A+B+C=\pi$, prove that

$$\cos(A)+\cos(B)+\cos(C) = 1+4\cdot\sin\left(\frac{A}{2}\right)\cdot\sin\left(\frac{B}{2}\right)\cdot\sin\left(\frac{C}{2}\right)$$

Ans: L.H.S.

$$= \cos(A)+\cos(B)+\cos(C)$$

$$= \cos(A)+\left\{2\cdot\cos\left(\frac{B+C}{2}\right)\cdot\cos\left(\frac{B-C}{2}\right)\right\}$$

$$\left\{\begin{array}{l} \because A+B+C=\pi \\ \Rightarrow \dfrac{A+B+C}{2}=\dfrac{\pi}{2} \\ \Rightarrow \dfrac{B+C}{2}=\left(\dfrac{\pi}{2}-\dfrac{A}{2}\right) \end{array}\right\}$$

White Page

Date

Page

$$= \cos(A) + \left\{ 2\cdot\cos\left(\frac{\pi - A}{2}\right)\cdot\cos\left(\frac{B-C}{2}\right) \right\}$$

$$= \cos(A) + \left\{ 2\cdot\sin\left(\frac{A}{2}\right)\cdot\cos\left(\frac{B-C}{2}\right) \right\}$$

$$= 1 - 2\cdot\sin^2\left(\frac{A}{2}\right) + \left\{ 2\cdot\sin\left(\frac{A}{2}\right)\cdot\cos\left(\frac{B-C}{2}\right) \right\}$$

$$= 2\cdot\sin\left(\frac{A}{2}\right)\left\{ -\sin\frac{A}{2} + \cos\left(\frac{B-C}{2}\right) \right\} + 1$$

$$= 2\cdot\sin\left(\frac{A}{2}\right)\cdot\left\{ \cos\left(\frac{-B-C}{2}\right) + \cos\left(\frac{B-C}{2}\right) \right\} + 1$$

$$= 2\cdot\sin\left(\frac{A}{2}\right)\cdot\left\{ 2\cdot\cos\left(\frac{-C}{2}\right)\cdot\cos\left(\frac{-B}{2}\right) \right\} + 1$$

$$= 4\cdot\sin\left(\frac{A}{2}\right)\cdot\cos\left(\frac{B}{2}\right)\cdot\cos\left(\frac{C}{2}\right) + 1$$

= R.H.S. proved

Q. If $A+B+C = \pi$, prove that :-

$$\frac{\sin(2A) + \sin(2B) + \sin(2C)}{\sin(A) + \sin(B) + \sin(C)} = 8\cdot\sin\left(\frac{A}{2}\right)\cdot\sin\left(\frac{B}{2}\right)\cdot\sin\left(\frac{C}{2}\right)$$

Ans: L.H.S.

1st part

$= \sin(2A) + \sin(2B) + \sin(2C)$

$= 2\sin A \cdot \cos A + 2\sin(B+C)\cdot\cos(B-C)$

$\left\{ \because A+B+C = \pi \Rightarrow (B+C) = \pi - A \right\}$

$= 2\sin A \cdot \cos A + 2\sin(\pi - A)\cdot\cos(B-C)$

$= 2\sin A \cdot \cos A + 2\sin(A)\cdot\cos(B-C)$

$= 2\sin A \left\{ \cos(A) + \cos(B-C) \right\}$

$\left\{ \because A+B+C = \pi \quad A = \pi - (B+C) \right\}$

$= 2\sin(A)\left\{ \cos(\pi - (B+C)) + \cos(B-C) \right\}$

$= 2\sin(A)\left\{ \cos(B+C) + \cos(B-C) \right\}$

$= 2\sin(A) \times \left\{ \cos(B+C) - \cos(C-B) \right\}$

$= 2\sin(A)\left\{ 2\sin(B)\cdot\sin(C) \right\}$

$= 4\cdot\sin(A)\cdot\sin(B)\cdot\sin(C)$

White Page

Date
Page

$$= 4 \cdot x \left\{ 2 \cdot \sin\left(\frac{A}{2}\right) \cdot \cos\left(\frac{A}{2}\right) \right\} x \left\{ 2 \sin\left(\frac{B}{2}\right) \cdot \cos\left(\frac{B}{2}\right) \right\} \left(\frac{x}{2}\right)$$

$$x \left\{ 2 \cdot \sin\left(\frac{C}{2}\right) \cdot \cos\left(\frac{C}{2}\right) \right\}$$

$$= 32 \cdot \sin\left(\frac{A}{2}\right) \cdot \cos\left(\frac{A}{2}\right) \cdot \sin\left(\frac{B}{2}\right) \cdot \cos\left(\frac{B}{2}\right) \cdot$$

$$\sin\left(\frac{C}{2}\right) \cdot \cos\left(\frac{C}{2}\right)$$

II nd part

$$= \sin(A) + \sin(B) + \sin(C)$$

$$= \sin(A) + \left\{ 2 \cdot \sin\left(\frac{B+C}{2}\right) \cdot \cos\left(\frac{B-C}{2}\right) \right\}$$

$$= \sin(A) + \left\{ 2 \cdot \sin\left(\frac{\pi - A}{2}\right) \cdot \cos\left(\frac{B-C}{2}\right) \right\}$$

$$\left\{ \begin{aligned} &\because A+B+C = \pi \\ &\Rightarrow \frac{A+B+C}{2} = \frac{\pi}{2} \\ &\Rightarrow \frac{B+C}{2} = \frac{\pi}{2} - \left(\frac{A}{2}\right) \end{aligned} \right\}$$

$$= \sin(A) + \left\{ 2 \cdot \cos\left(\frac{A}{2}\right) \cdot \cos\left(\frac{B-C}{2}\right) \right\}$$

$$= 2\cdot \sin\left(\frac{A}{2}\right)\cdot \cos\left(\frac{A}{2}\right) + \left\{2\cdot \cos\left(\frac{A}{2}\right)\cdot \cos\left(\frac{B-C}{2}\right)\right\}$$

$$= 2\cdot \cos\left(\frac{A}{2}\right)\left\{\sin\left(\frac{A}{2}\right) + \cos\left(\frac{B-C}{2}\right)\right\}$$

$$\left\{\begin{array}{l} \because A+B+C = \pi \\ \Rightarrow \frac{A+B+C}{2} = \frac{\pi}{2} \\ \Rightarrow \frac{A}{2} = \frac{\pi}{2} - \left(\frac{B+C}{2}\right) \end{array}\right\}$$

$$= 2\cdot \cos\left(\frac{A}{2}\right)\left\{\sin\left(\frac{\pi}{2} - \left(\frac{B+C}{2}\right)\right) + \cos\left(\frac{B-C}{2}\right)\right\}$$

$$= 2\cdot \cos\left(\frac{A}{2}\right)\left\{\cos\left(\frac{B+C}{2}\right) + \cos\left(\frac{B-C}{2}\right)\right\}$$

$$= 2\cdot \cos\left(\frac{A}{2}\right)\left\{2\cdot \cos\left(\frac{B}{2}\right)\cdot \cos\left(\frac{C}{2}\right)\right\}$$

$$= 4\cdot \cos\left(\frac{A}{2}\right)\cdot \cos\left(\frac{B}{2}\right)\cdot \cos\left(\frac{C}{2}\right)$$

Therefore,

L.H.S.

$$= \frac{\text{I}^{\text{st}} \text{ part}}{\text{II}^{\text{nd}} \text{ part}}$$

$$= \frac{\cancel{32}\,8 \cdot \sin\left(\frac{A}{2}\right) \cdot \cancel{\cos\left(\frac{A}{2}\right)} \cdot \sin\left(\frac{B}{2}\right) \cdot \cancel{\cos\left(\frac{B}{2}\right)} \cdot \sin\left(\frac{C}{2}\right) \cdot \cancel{\cos\left(\frac{C}{2}\right)}}{\cancel{4 \cdot \cos\left(\frac{A}{2}\right) \cdot \cos\left(\frac{B}{2}\right) \cdot \cos\left(\frac{C}{2}\right)}}$$

$$= 8 \cdot \sin\left(\frac{A}{2}\right) \cdot \sin\left(\frac{B}{2}\right) \cdot \sin\left(\frac{C}{2}\right)$$

= R.H.S. proved.

Q. If $A+B+C = \pi$, prove that

$$\sin(B+C-A) + \sin(C+A-B) - \sin(A+B-C) = 4 \cdot \cos(A) \cdot \cos(B) \cdot \cos(C)$$

Ans: L.H.S.

$$= \sin(B+C-A) + \sin(C+A-B) - \sin(A+B-C)$$

$$= 2 \cdot \sin(C) \cdot \cos(B-A) - \sin(A+B-C)$$

$$\left\{ \begin{array}{l} \because A+B+C = \pi \\ \Rightarrow B+A = \pi - C \end{array} \right\}$$

$$= 2 \cdot \sin C \cdot \cos(B-A) - \sin(\pi - C - C)$$

$$= 2 \cdot \sin C \cdot \cos(B-A) - \sin 2C$$

$$= 2 \cdot \sin C \cdot \cos(B-A) - 2 \cdot \sin C \cdot \cos C$$

$$= 2 \cdot \sin C \left\{ \cos(B-A) - \cos C \right\}$$

$$= 2 \cdot \sin C \left\{ 2 \sin\left(\frac{B-A+C}{2}\right) \cdot \sin\left(\frac{C-B+A}{2}\right) \right\}$$

$$= 2 \cdot \sin C \cdot \left\{ 2 \sin\left(\frac{\pi - A - A}{2}\right) \cdot \sin\left(\frac{\pi - B - B}{2}\right) \right\}$$

$$= 4 \cdot \cos A \cdot \cos B \cdot \sin C$$

$$= R \cdot H \cdot S$$